

# An Approximate Model for Determining the Resistance of a Hemispherical Ground Electrode Placed on a Non-homogeneous Truncated Cone

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*Abstract: In this paper we present an approximate analytically oriented approach for determining the resistance of a hemi-spherically-shaped ground electrode placed on the top of a mountain. The mountain is modelled as a non-homogeneous truncated cone consisting of two homogeneous domains with different specific conductivities values. The given procedure extends the existing, previously proposed procedure in which the mountain was modelled as a homogeneous truncated cone. The procedure, proposed in this paper, includes application of the Estimation method, based on idea of finding arithmetic mean of maximal and minimal resistance value. The obtained results are validated and compared with those obtained using the COMSOL program package, based on the finite element method.*

*Keywords: Estimation method; grounding system; non-homogeneous soil; resistance; hemispherical electrode*

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## 1 Introduction

The resistance of the grounding electrode is its most important characteristic and therefore it is of the utmost importance to develop methods for its estimation. This value is influenced by electrode geometry, conductor characteristics, soil structure related to the specific conductivity structure and physical shape of the surrounding ground. The approach of modelling ground as homogeneous half space of flat ground has been used a decades ago [1-2]. The similar is with the procedures which include non-homogeneous ground modeled as multi-layered [3-4], sectoral

[5], semi-spherically [6-7] or semi-cylindrically shaped domain [8]. All these approaches include using of various appropriate numerical methods [9-14]. Very often, facilities having grounding systems as a necessary part are placed at mountains or hills (e.g. antenna towers, wind turbines, etc.) [15-19]. Usually, those terrains are of low specific conductivity. Research dealing with the problem of characterization of grounding systems installed in such places is not so common. An interesting procedure for analysis of a hemispherical ground electrode placed at the top of a hill is proposed in [20]. It is based on the idea of approximating a mountain with a homogeneous truncated cone, while the hemispherical electrode surface is modelled as calotte. These assumptions allow generating approximate analytical expression for the resistance value. In [21], the above-mentioned procedure is improved by assuming current density distribution in two different forms, depending on the observed domain. In this paper, an extension of the approach from [21] is proposed. It offers the possibility of modelling a hill as a non-homogeneous domain consisting of two homogeneous areas, each having different electrical characteristics. The Estimation method application [8, 22-24] is part of the procedure described in this paper. This method is based on the idea of determining the desired approximate value as an arithmetic mean of the upper and lower limits of the interval where the corresponding solution is expected to be. The COMSOL program package, based on the finite element method, is used to validate the results. The obtained results and the data analysis performed during validation suggest that the proposed, relatively simple approach is satisfactory accurate, especially for practical engineering purposes. There is no need for any integration involved in the procedure which reduces the resistance determination to an arithmetic equation. The proposed approach can also be extended to a hill modelled as a truncated cone consisting of three or more domains of different specific conductivity values. Also, the procedure can be used for both, flat or semi-spherically-shaped boundary surface between domains of different specific conductivity values.

## 2 Problem Description and Solution Procedure

### 2.1 Problem Description

The hemispherical ground electrode placed at the top of a hill approximated with a truncated cone is observed, as depicted in Figure 1. The cone consists of two homogeneous domains having specific conductivities  $\sigma_1$  and  $\sigma_2$ . The radius of the electrode is  $r_0$  and cone base radius is  $r_t$ . Other geometry parameters from Figure 1 are self-explanatory. In this paper, proposed solution for the structure from Figure 1 is based, as it has been already emphasized, on approaches from

[20] and [21]. However, the chosen model of non-homogeneous truncated cone is more complex and realistic structure related to those from [20] and [21]. One could expect that in general case ground structure is non-homogeneous and using the model from Figure 1 is a good way to take ground non-homogeneity into account.

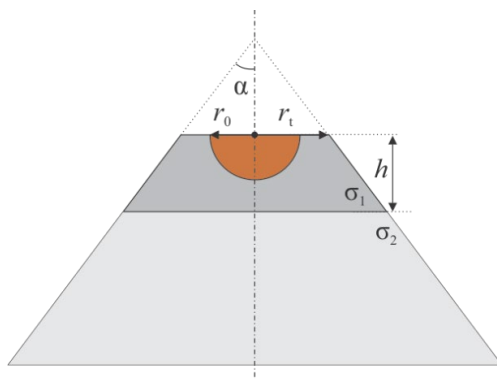


Figure 1

The hemispherical ground electrode at the top of a hill

## 2.2 Solution Procedure

Since approaches from [20] and [21] are the bases of solving of the problem illustrated in Figure 1, they will be briefly described in this chapter, before presentation of extended procedure applied on model from Figure 1.

### 2.2.1 Basic Procedure

In [20], the problem of hemispherical grounding electrode having radius  $r_0$ , placed at the top of the truncated homogeneous cone of a specific conductivity  $\sigma$  (Figure 2) is analysed and corresponding analytical solution for the low-frequency resistance is derived. A brief description of this procedure is given in this chapter.

The essence of the procedure given in [20] is approximation of the hemispherical electrode by a spherical sector of the radius  $R_1$  (the center of spherical sector is at the fictitious peak of the cone), Figure 2. The parameters  $d$  and  $\alpha$  are marked in the Figure 2. It is assumed that electrode is fed by the quasi-stationary current  $I$ .

From Figure 2 follows:

$$R_1 = r_0 + d, \quad d = r_0 \cot \alpha \quad \text{and} \quad R_1 = r_0 (1 + \cot \alpha). \quad (1)$$

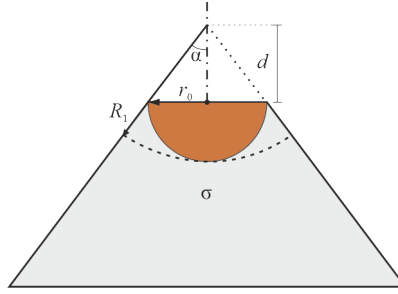


Figure 2

The hemispherical ground electrode at the top of a homogeneous truncated cone

A surface area of a spherical sector of radius  $R$  (coordinate origin for this coordinate coincide with the fictitious pick of the cone) is  $S = \Omega R^2$ , where  $\Omega = 2\pi(1 - \cos \alpha)$  is solid angle. Hence, surface area is

$$S = R^2 2\pi(1 - \cos \alpha). \quad (2)$$

Symmetry of the approximate geometry has as a consequence that current density depends only on radial coordinate  $r$ , i.e

$$\vec{J} = \frac{I}{S} \hat{R} = \frac{I}{R^2 2\pi(1 - \cos \alpha)} \hat{r}, \quad R_1 < R < \infty. \quad (3)$$

Now, using the local form of Ohm's law,  $\vec{J} = \sigma \vec{E}$ , where  $\vec{E}$  is electric field vector, the approximate potential of the electrode surface is

$$\varphi_s = \int_{R_1}^{\infty} \frac{J}{\sigma} dR = \int_{R_1}^{\infty} \frac{I}{2\pi\sigma(1 - \cos \alpha)R^2} dR = \frac{I}{2\pi\sigma(1 - \cos \alpha)R_1}. \quad (4)$$

Considering that  $R_1 = r_0(1 + \cot \alpha)$ , the electrode surface potential is

$$\varphi_s = \frac{I}{2\pi\sigma(1 - \cos \alpha)(1 + \cot \alpha)r_0}. \quad (5)$$

Now, resistance of the hemispherical grounding electrode is

$$R_e = \frac{\varphi_s}{I} = \frac{1}{2\pi\sigma(1 - \cos \alpha)(1 + \cot \alpha)r_0}. \quad (6)$$

### 2.2.2 Improved Basic Procedure

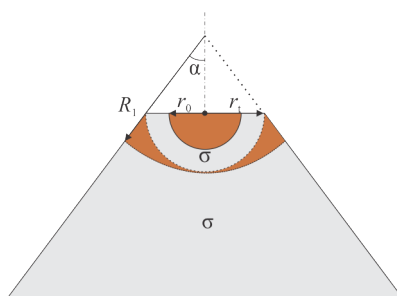


Figure 3

The hemispherical ground electrode at the top of a homogeneous truncated cone

The basic procedure from [20] is extended in [21] and applied to the problem of the hemispherical ground electrode having radius smaller than the radius of the cone basis, Figure 3. The electrode is fed in the center by quasi-stationary current  $I$ . The truncated cone has specific conductivity  $\sigma$ , while the radii of the hemispherical electrode and upper cone base are  $r_0$  and  $r_t$  respectively ( $r_0 < r_t$ ). This approach includes approximation of the current field with two different expressions.

Firstly, in area defined by  $r_0 < r < r_t$ , where  $r$  is radial coordinate having origin at the center of the hemispherical electrode, the current density vector is assumed as

$$\vec{J}_1 = \frac{I}{2\pi r^2} \hat{r}, \quad r_0 < r < r_t. \quad (7)$$

As in [20], a spherical sector of the radius  $R_1$  (Figure 3) is introduced into the model. Below this sector, defined by  $R_1 < R < \infty$ , where  $R$  is radial coordinate having origin at the fictitious pick of the cone, for the current density vector the following expression is used

$$\vec{J}_2 = \frac{I}{2\pi(1-\cos\alpha)R^2} \hat{R}, \quad R_1 < R < \infty \quad (8)$$

Now, the potential of the electrode can be determined as

$$\varphi_s = \int_{r_0}^{r_t} \frac{J_1}{\sigma} dr + \int_{R_1}^{\infty} \frac{J_2}{\sigma} dR = \int_{r_0}^{r_t} \frac{I}{2\pi\sigma r^2} dr + \int_{R_1}^{\infty} \frac{I}{2\pi\sigma(1-\cos\alpha)R^2} dR. \quad (9)$$

From expression (9) follows

$$\varphi_s = \frac{1}{2\pi\sigma} \left[ \frac{1}{r_0} - \frac{1}{r_t} + \frac{1}{(1-\cos\alpha)R_1} \right]. \quad (10)$$

Consequently, the resistance of the hemispherical electrode from Figure 3 is

$$R_c = \frac{\varphi_s}{I} = \frac{1}{2\pi\sigma} \left[ \frac{1}{r_0} - \frac{1}{r_t} + \frac{1}{(1-\cos\alpha)R_1} \right]. \quad (11)$$

### 2.2.3 Solution Procedure for the system from Figure 1

In order to approximately determine resistance of the hemispheric electrode from Figure 1, the model depicted in Figure 4 will be analysed. The truncated cone consists of two homogeneous domains having specific conductivities  $\sigma_1$ , i.e.  $\sigma_2$ . The electrode is fed by quasi-stationary current  $I$ . The radius of the electrode is  $r_0$  and cone basis radius is  $r_t$ . The boundary surface between two domains is the spherical sector of the radius  $R_2$ .

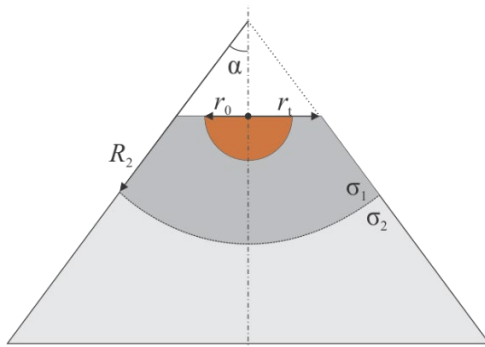


Figure 4

Hill approximated with two-domain truncated cone

A part of the system from Figure 4, consisting of hemispherical electrode and domain having specific conductivity value  $\sigma_1$ , is approximated as in [21] (and explained in 2.2.2), as it is shown in Figure 5. As proposed in [21], described part has been replaced with a hemispherical electrode placed in a shell with boundary of radius  $R_1$ . As already written,  $r_0$  and  $r_t$  are the distances from the centre of the hemisphere, while  $R_1$  and  $R_2$  are distances from the origin positioned at the fictitious top of the cone. As in [21], the current field in a hemispherical shell around the electrode is assumed as radial, having a current density

$$\vec{J}_1 = \frac{I}{2\pi r^2} \hat{r}, \quad r_0 < r < r_t. \quad (12)$$

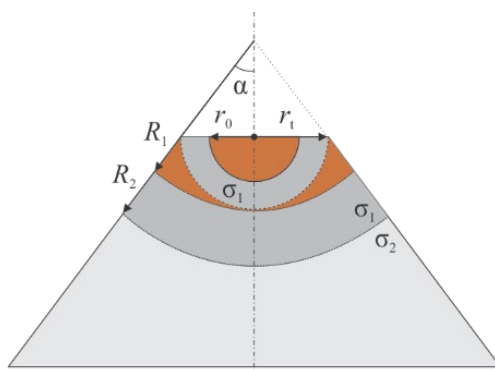


Figure 5

Illustration of the procedure from [21] for part of the system of specific conductivity  $\sigma_1$

The current field in the rest of the domain of specific conductivity  $\sigma_1$  is assumed as radial (following the approach from [21]), related to the fictitious top of the cone. It can be characterized by current density vector [21],

$$\vec{J}_2 = \frac{I}{2\pi(1-\cos\alpha)R^2} \hat{R}, \quad R_1 < R < R_2. \quad (13)$$

In expression (13),  $R$  corresponds to the radial distance from the top of the cone, while  $\hat{R}$  corresponds to radial ort. The same expression can also be applied for current density vector in the hill domain of specific conductivity  $\sigma_2$  (defined with  $R_2 < R < \infty$ ), based on the boundary condition for normal component of quasi-stationary current density vector, for  $R = R_2$ .

Now, the potential of the electrode surface related to the referent point placed on a large distance from the electrode, based on previous assumptions, can be determined as

$$\varphi_s = \int_{r_0}^{r_1} \frac{J_1}{\sigma_1} dr + \int_{R_1}^{R_2} \frac{J_2}{\sigma_1} dR + \int_{R_2}^{\infty} \frac{J_2}{\sigma_2} dR. \quad (14)$$

In the previous expressions,  $dr$  and  $dR$  are differentials of the radial coordinates defined in the text above.

Now, using Ohm's law and equations (12)-(14), the following approximate expression for the electrode potential is obtained.

$$\varphi_s = \int_{r_0}^{r_1} \frac{I}{2\pi\sigma_1 r^2} dr + \int_{R_1}^{R_2} \frac{I}{2\pi\sigma_1(1-\cos\alpha)R^2} dR + \int_{R_2}^{\infty} \frac{I}{2\pi\sigma_1(1-\cos\alpha)R^2} dR. \quad (15)$$

From (15), the resistance of the hemispherical electrode from Figure 3 is,

$$R_e = \frac{\varphi_s}{I} = \frac{1}{2\pi\sigma_1} \left\{ \frac{1}{r_0} - \frac{1}{r_t} + \frac{1}{(1-\cos\alpha)} \left[ \frac{1}{R_1} - \frac{1}{R_2} \right] \right\} + \frac{1}{2\pi\sigma_2} \frac{1}{(1-\cos\alpha)R_2}. \quad (16)$$

where, from Figure 5 follows that

$$R_1 = (1 + \cot\alpha)r_t. \quad (17)$$

Finally, using equation (16) and the Estimation method [8, 22-24], it is possible to form an approximate expression for determining the ground electrode's resistance from Figure 1. In Figure 6 are labelled upper ( $R_{2e}$ ) and lower ( $R_{2i}$ ) values of boundary surface radii. The approximate resistance of the system is determined as arithmetic mean of the resistance values obtained for Figure 6,

$$R_{2i} = (1 + \cot\alpha)r_t \text{ and } R_{2e} = \frac{h + r_t \cot\alpha}{\cos\alpha}. \quad (18)$$

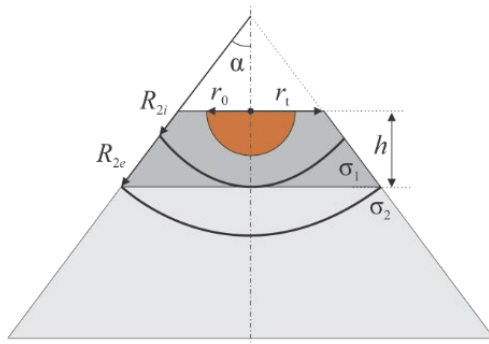


Figure 6

Estimation method application

Now, for  $R_2 = R_{2i}$  the value of the electrode resistance  $R_e = R_{ei}$  is

$$R_{ei} = \frac{1}{2\pi\sigma_1} \left\{ \frac{1}{r_0} - \frac{1}{r_t} + \frac{1}{(1-\cos\alpha)} \left[ \frac{1}{R_1} - \frac{1}{R_{2i}} \right] \right\} + \frac{1}{2\pi\sigma_2} \frac{1}{(1-\cos\alpha)R_{2i}}. \quad (19)$$

For  $R_2 = R_{2e}$  obtains electrode resistance  $R_e = R_{ee}$ , i.e.

$$R_{ee} = \frac{1}{2\pi\sigma_1} \left\{ \frac{1}{r_0} - \frac{1}{r_t} + \frac{1}{(1-\cos\alpha)} \left[ \frac{1}{R_1} - \frac{1}{R_{2e}} \right] \right\} + \frac{1}{2\pi\sigma_2} \frac{1}{(1-\cos\alpha)R_{2e}}. \quad (20)$$

The approximate resistance value is obtained as the mean value of  $R_{2i}$  and  $R_{2e}$



$$R_{eap} = \frac{R_{ei} + R_{ee}}{2}, \text{ i.e.} \quad (21)$$

$$R_{eap} = \frac{1}{2\pi\sigma_1} \left\{ \frac{1}{r_0} - \frac{1}{r_t} + \frac{1}{(1-\cos\alpha)} \left[ \frac{1}{R_1} - \frac{R_{2e} + R_{2i}}{2R_{2e}R_i} \right] \right\} + \frac{1}{2\pi\sigma_2(1-\cos\alpha)} \frac{R_{2e} + R_{2i}}{2R_{2e}R_i}. \quad (22)$$

### 3 Results

The described method is applied for  $\sigma_1 = 0.01 \text{ S/m}$  and  $r_0 = 5 \text{ m}$ , while the rest of the parameters from Figure 1 take the following values:  $\alpha \in \{45^\circ, 50^\circ, 55^\circ, 60^\circ\}$ ,  $r_i \in \{5 \text{ m}, 10 \text{ m}, 15 \text{ m}\}$ ,  $h \in \{5 \text{ m}, 10 \text{ m}\}$  and  $\sigma_2/\sigma_1 \in \{0.5, 5\}$ . The values of the parameters have been selected based on [20]-[22]. The obtained results ( $R_{eap}$ ) are validated with the values obtained from the COMSOL program package application ( $R_e$ ). Number of boundary elements used during the simulation is 18562, while total number of elements is 308473. Electric potential distribution obtained in COMSOL for  $\alpha=45^\circ$ ,  $r_0=5 \text{ m}$ ,  $r_i=10 \text{ m}$ ,  $h=20 \text{ m}$ ,  $\sigma_1=0.01 \text{ S/m}$  and  $\sigma_2=0.0001 \text{ S/m}$  is shown in Figure 7.

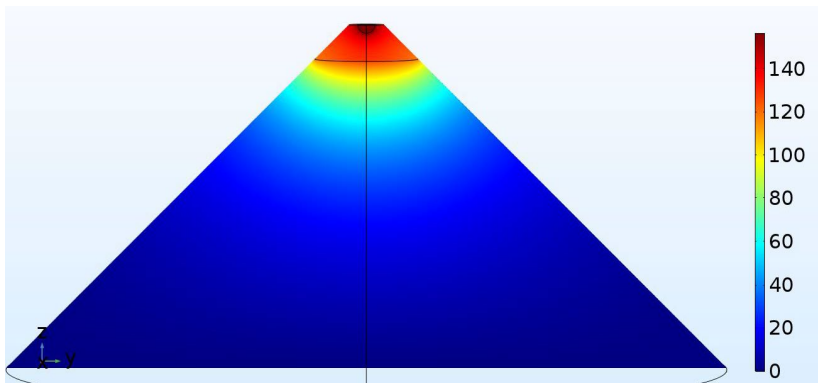


Figure 7

Electric potential distribution for  $\alpha=45^\circ$ ,  $r_0=5 \text{ m}$ ,  $r_i=10 \text{ m}$ ,  $h=20 \text{ m}$ ,  $\sigma_1=0.01 \text{ S/m}$  and  $\sigma_2=0.0001 \text{ S/m}$

The results ( $R_{eap}$ ,  $R_e$  and relative error) for  $\alpha \in \{45^\circ, 50^\circ, 55^\circ, 60^\circ\}$  are given in Tables 1-4, respectively. Graphics shown in Figures 8-11 correspond to Tables 1-4 respectively and contain relative error versus angle  $\alpha$  value.

The maximum deviation of the presented results (Tables 1-4) is 15.7%, while the standard deviation value is 4.541% (based on the results from Tables 1-4), related to the median value of 5.001%.

Table 1  
The results for  $\alpha=45^\circ$

| $r_t$ [m] | $h$ [m] | $\sigma_2 / \sigma_1$ | $R_{e\ ap}$ [ $\Omega$ ] | $R_e$ [ $\Omega$ ] | Relative error [%] |
|-----------|---------|-----------------------|--------------------------|--------------------|--------------------|
| 5         | 10      | 0.5                   | 85.260                   | 86.979             | 1.977046           |
|           |         | 5                     | 29.602                   | 35.110             | 15.68705           |
|           | 20      | 0.5                   | 72.891                   | 74.731             | 2.461634           |
|           |         | 5                     | 39.497                   | 45.459             | 13.1162            |
| 10        | 10      | 0.5                   | 66.276                   | 65.490             | 1.199309           |
|           |         | 5                     | 24.532                   | 26.905             | 8.816484           |
|           | 20      | 0.5                   | 58.545                   | 57.621             | 1.60396            |
|           |         | 5                     | 30.717                   | 33.770             | 9.040823           |
| 15        | 10      | 0.5                   | 57.886                   | 56.317             | 2.785968           |
|           |         | 5                     | 24.492                   | 25.113             | 2.47417            |
|           | 20      | 0.5                   | 52.585                   | 50.564             | 3.9976             |
|           |         | 5                     | 28.732                   | 30.455             | 5.657585           |

Table 2  
The results for  $\alpha=50^\circ$

| $r_t$ [m] | $h$ [m] | $\sigma_2 / \sigma_1$ | $R_{e\ ap}$ [ $\Omega$ ] | $R_e$ [ $\Omega$ ] | Relative error [%] |
|-----------|---------|-----------------------|--------------------------|--------------------|--------------------|
| 5         | 10      | 0.5                   | 74.233                   | 76.341             | 2.761091           |
|           |         | 5                     | 27.828                   | 32.805             | 15.16972           |
|           | 20      | 0.5                   | 63.578                   | 65.790             | 3.361815           |
|           |         | 5                     | 36.352                   | 41.843             | 13.12202           |
| 10        | 10      | 0.5                   | 60.041                   | 59.690             | 0.589018           |
|           |         | 5                     | 24.222                   | 26.182             | 7.485269           |
|           | 20      | 0.5                   | 53.032                   | 52.567             | 0.885395           |
|           |         | 5                     | 29.830                   | 32.524             | 8.284733           |
| 15        | 10      | 0.5                   | 53.575                   | 52.558             | 1.934003           |
|           |         | 5                     | 24.409                   | 24.754             | 1.3937             |
|           | 20      | 0.5                   | 48.602                   | 47.136             | 3.110419           |
|           |         | 5                     | 28.387                   | 29.880             | 4.99731            |

Table 3  
The results for  $\alpha=55^\circ$

| $r_t$ [m] | $h$ [m] | $\sigma_2 / \sigma_1$ | $R_{e\ ap}$ [ $\Omega$ ] | $R_e$ [ $\Omega$ ] | Relative error [%] |
|-----------|---------|-----------------------|--------------------------|--------------------|--------------------|
| 5         | 10      | 0.5                   | 65.655                   | 67.773             | 3.125576           |
|           |         | 5                     | 26.504                   | 30.907             | 14.2475            |
|           | 20      | 0.5                   | 56.400                   | 58.725             | 3.95919            |

|    |    |     |        |        |          |
|----|----|-----|--------|--------|----------|
|    |    | 5   | 33.908 | 38.896 | 12.82292 |
| 10 | 10 | 0.5 | 55.139 | 54.986 | 0.278231 |
|    |    | 5   | 24.050 | 25.599 | 6.04808  |
|    | 20 | 0.5 | 48.743 | 48.520 | 0.459313 |
|    |    | 5   | 29.167 | 31.494 | 7.387051 |
| 15 | 10 | 0.5 | 50.178 | 49.445 | 1.48177  |
|    |    | 5   | 24.397 | 24.470 | 0.295781 |
|    | 20 | 0.5 | 45.482 | 44.358 | 2.535321 |
|    |    | 5   | 28.154 | 29.398 | 4.233505 |

Table 4  
The results for  $\alpha=60^{\circ}$

| $r_1$ [m] | $h$ [m] | $\sigma_2 / \sigma_1$ | $R_{e,ap}$ [ $\Omega$ ] | $R_e$ [ $\Omega$ ] | Relative error [%] |
|-----------|---------|-----------------------|-------------------------|--------------------|--------------------|
| 5         | 10      | 0.5                   | 58.885                  | 60.850             | 3.228145           |
|           |         | 5                     | 25.540                  | 29.320             | 12.89244           |
|           | 20      | 0.5                   | 50.791                  | 53.017             | 4.19838            |
|           |         | 5                     | 32.015                  | 36.424             | 12.10466           |
| 10        | 10      | 0.5                   | 51.231                  | 51.113             | 0.229494           |
|           |         | 5                     | 23.988                  | 25.124             | 4.522457           |
|           | 20      | 0.5                   | 45.358                  | 45.223             | 0.299508           |
|           |         | 5                     | 28.685                  | 30.641             | 6.383074           |
| 15        | 10      | 0.5                   | 47.468                  | 46.885             | 1.243428           |
|           |         | 5                     | 24.439                  | 24.241             | 0.815542           |
|           | 20      | 0.5                   | 43.004                  | 42.071             | 2.216863           |
|           |         | 5                     | 28.010                  | 29.004             | 3.427549           |

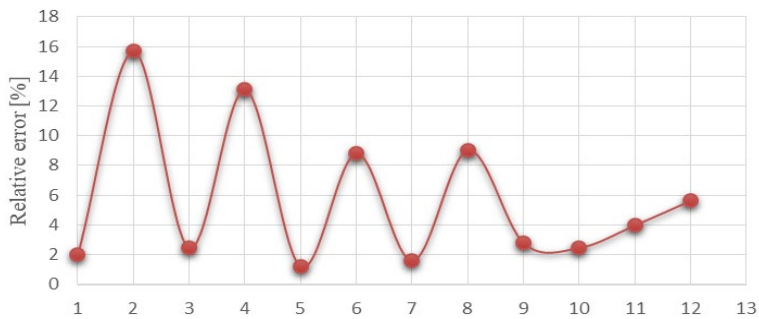


Figure 8  
Relative error for different samples when  $\alpha=45^{\circ}$  (Table 1)

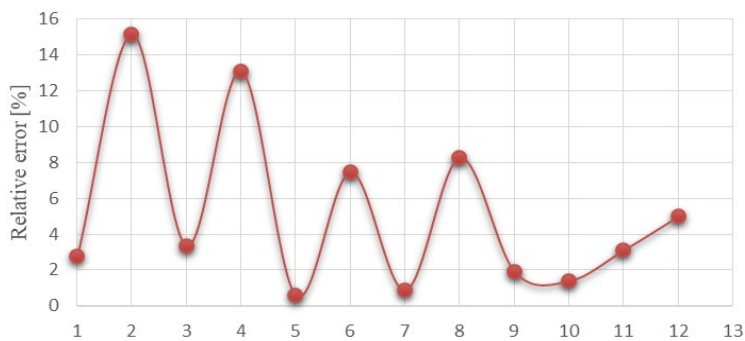


Figure 9  
Relative error for different samples when  $\alpha=50^\circ$  (Table 2)

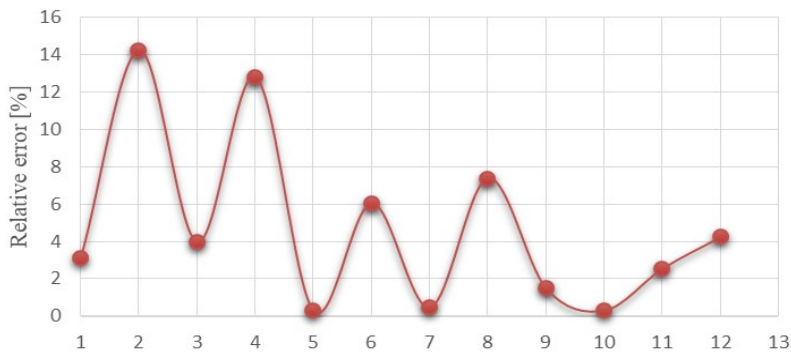


Figure 10  
Relative error for different samples when  $\alpha=55^\circ$  (Table 3)

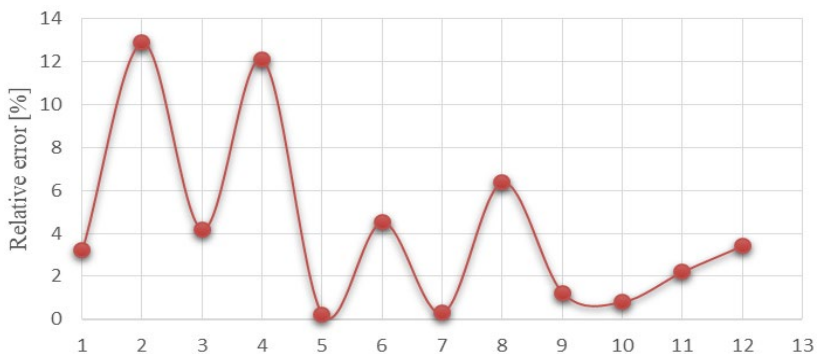


Figure 11  
Relative error for different samples when  $\alpha=60^\circ$  (Table 4)

## Discussion and Conclusions

An approximate analytical procedure for determining the resistance of a hemispherical electrode, placed on top of a mountain, is presented. The mountain is modelled as truncated cone with two domains. Proposed procedure is a kind of extension of the methods given in [20] and [21]. Based on the obtained results, it can be concluded that the proposed approach is satisfactorily accurate, especially for engineering applications. The method does not involve any type of integration and reduces resistance determination to a simple arithmetic equation. It can also be extended to a hill modelled as a truncated cone consisting of three or more domains of different specific conductivity values.

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