Optimized Trapezoidal Stiffened Plates under Uniaxial Compression with a Sudden, Rapidly Applied Pressure

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Abstract: This paper is devoted to the forced vibration analysis of optimized trapezoidal stiffened plates with simple supported conditions on the four edges of the base plate. The purpose of the finite element analysis is to investigate the transient forced vibrations of stiffened structures subjected to uniaxial compression due to the reason of rapidly applied pressure over their base plates, thereby identifying potentially dangerous cases and minimizing the possibility of failure. In this study, the numerical analysis is performed for such a design of this kind of welded plates which have already been optimized for lateral pressure and uniaxial compression as static loadings. The objective function of the optimization to be minimized performed with the Excel Solver program is the cost function which contains material and fabrication costs for Gas Metal Arc Welding (GMAW) welding technology. The eigenvalue extraction is used to calculate the natural frequencies and mode shapes based on the Lanczos iteration method then the transient response is determined using the modal superposition method from the first few mode shapes. The welded structure is made of two grades of steel, which are described with different yield stress while all other material properties of steel remain the same.

Keywords: trapezoidal stiffener; FEA; modal analysis; transient forced vibration

1 Introduction

Stiffened plates are commonly used in various engineering applications because they are able to resist buckling and provide additional stability and rigidity to the structure. Welded stiffened plates are an important component of many engineering structures, including shipbuilding, offshore platforms and civil engineering, such as, bridges. An assessment of their free and forced responses is also generally very important for the safe and rational structural design of these members of structures. These plates are made of thin metal sheets that are reinforced by stiffeners, which are typically placed parallel to the plate's surface, and they are formed by welding stiffeners on bare plates. The stiffeners provide increased strength and stability, enabling the plate to better withstand loads such as bending, torsion, and buckling. The shape of plates can be square rectangular, circular, trapezoidal, etc. They can be stiffened in one or two directions with stiffeners of flat, L, box, trapezoidal or other shapes. The central deflection of simple supported stiffened plates subjected to transverse and axial load can be estimated, as well as the influence of parameters such as the number of longitudinal and transverse stiffeners and the ratio between their height and thickness. In recent years, there has been a growing interest in improving the design of stiffened plates to make them lighter, stronger and more cost-effective.

To this end, researchers have been exploring new materials, new fabrication techniques, and new design strategies, which remain a widely researched topic. Several researchers have studied the dynamic responses of rectangular plate systems having different sets of edge constraints and loading cases [1] [2]. The refined empirical formulation proposed to predict the ultimate strength performance or ultimate limit state of flat-bar type steel stiffened panels under longitudinal compression [3]. Stability issues have a significant impact on welded structures in terms of local, general and torsional buckling [4].

Nowadays, an important field of application of stiffened plates is ship architecture and offshore engineering, where they are basic structural members. Various loads on a ship's structure are complex phenomena that have been the subject of extensive research in the field of naval architecture. The design of a ship's hull, as well as its stability and strength, are greatly affected by wave loads, which arise from the interaction between the ship and the ocean waves. The intensity and frequency of wave loads depend on various factors such as the ship's speed, wave height, and frequency, as well as the ship's geometry, displacement, and type of loading. The understanding of wave loads is important for the design and safe operation of ships, as well as for the prediction of the response of ships to these loads, including structural deformation, slamming, and stress. The frequency response of plates with openings subjected to point excitation force and enforced acceleration at boundaries is analyzed by using developed in-house code for the mode superposition method [5]. Yang et. al. [6] dealt with the dynamic ultimate strength of ship bottom stiffened plates under uniaxial compression and lateral pressure. The dynamic ultimate strength of a tested specimen was calculated based on the nonlinear FEA.

The forced vibration analysis of optimized trapezoidal stiffened plates with simple supported conditions on the four edges of the base plate provides valuable insights into the transient dynamic behavior of the structure. Drawing the conclusion from finite element analysis, this circumstance greatly affects the result.

The optimization process used in this study ensures that the final design is both economically and structurally efficient, while the use of the Lanczos iteration method provides a reliable and efficient way to calculate the natural frequencies, mode shapes, and the modal superposition method is an effective technique for reducing the computation time when performing dynamic response analyses of linear structures like transient force vibration analyses [7]. Dynamic analysis of time response is widely used in various engineering problems to study the effect of vibrations on the members of structures [8] [9].

In the present study, the focus is on the forced vibration analysis of optimized trapezoidal stiffened plates with simple supported conditions on the four edges of the base plate. Forced vibration analysis is a critical aspect of structural engineering and is essential in ensuring the safety and reliability of structures subjected to dynamic loads. The aim of the finite element analysis is to investigate the transient forced vibrations of the stiffened structures subjected to uniaxial compression from sudden and rapidly applied pressure, over the base plates [10]. This study is essential in identifying potentially dangerous cases and eliminating the possibility of failure. The optimization process used in this study minimizes the cost function that contains material and fabrication costs for Gas Metal Arc Welding (GMAW) welding technology. The optimization was performed with the Excel Solver program to ensure that the final design is both economically and structurally efficient.

2 Optimum Design of Stiffened Plates

The structural optimization of different stiffened plates and shells has been worked out by Farkas and Jármai [11]. The optimum design of stiffened plates involves finding the optimal combination of plate and stiffener geometry, material properties, and loading conditions that meet the desired mechanical performance while minimizing weight.

The design of stiffened plates requires a detailed understanding of plate buckling, plate-stiffener interaction, and stress distribution. Plate buckling occurs when the plate experiences a compressive load that exceeds its critical buckling strength. The buckling behavior of a plate can be influenced by its geometry, boundary conditions, and material properties. Plate-stiffener interaction refers to the interaction between the plate and the stiffener, which can enhance the plate's resistance to buckling. The distribution of stress in a stiffened plate is influenced by the plate-stiffener interaction and the loading conditions.

The optimum design of stiffened plates can be performed using various numerical methods, such as finite element analysis (FEA), boundary element analysis (BEA), and multi-objective optimization algorithms. FEA and BEA allow for a detailed analysis of the plate-stiffener interaction and the stress distribution in the plate, while multi-objective optimization algorithms allow for an efficient search of the design space to find the optimal solution.

In the optimum design of stiffened plates, various design objectives must be considered, such as minimizing weight, maximizing strength, reducing deflection, and minimizing stress concentration. These design objectives may conflict with each other, and the optimum design must find a trade-off that satisfies all objectives. The trade-off can be performed using multi-objective optimization algorithms, such as genetic algorithms, particle swarm optimization, and ant colony optimization.

The material properties of the plate and the stiffener also play an important role in the optimum design of stiffened plates. The material properties, such as yield strength, elastic modulus, and Poisson's ratio, can influence the plate's resistance to buckling and the distribution of stress in the plate. The use of advanced materials, such as composite materials, can significantly improve the mechanical performance of stiffened plates, but also increase their complexity and cost. The composite structures are also a way to reduce the mass of the structure [12].

The boundary conditions of the plate and the stiffener also play a crucial role in the optimum design of stiffened plates. The boundary conditions, such as fixed-end, simple supported, and clamped, can influence the plate's buckling behavior and the distribution of stress in the plate. The boundary conditions must be carefully selected to ensure that the plate's buckling behavior and stress distribution are consistent with the desired mechanical performance.

In conclusion, the optimum design of stiffened plates is a complex process that involves finding the optimal combination of plate and stiffener geometry, material properties, and loading conditions that meet the desired mechanical performance while minimizing weight. The optimum design requires a detailed understanding of plate buckling, plate-stiffener interaction, and stress distribution, as well as the use of numerical methods and multi-objective optimization algorithms. The design objectives, material properties, and boundary conditions must also be carefully considered to ensure that the optimum design satisfies the desired mechanical performance.

With a better understanding of the optimum design of stiffened plates, engineers can design structures that are lighter, stronger more cost-effective and contribute to the development of sustainable/efficient engineering solutions [13].

In the following calculation, the base plate has a width of $B = 4000$ mm and a length of $L = 6000$ mm. Young’s modulus is $E = 2.1 \times 10^5$ MPa, material density is $\rho = 7.85 \times 10^{-9}$ t/mm3. The stiffeners are welded to the base plate with fillet welds to reinforce the plate. The design variables, the thicknesses of the base plate (t_F) and stiffener (t_S) and the number of ribs ($\varphi\text{-}1$). The numerical results for uniaxial compression and magnitude of lateral pressure are summarized in Tab. 1, in which the columns contain the optimized main geometrical data. The geometric designs of the structures, based on Table 1, were used for the transient vibration analysis in this paper, in which the pressure is considered to be suddenly applied. The objective of the forced vibration analysis is to investigate whether the optimization objective functions are also satisfied under dynamic effects.

Table 1
Optimum dimensions for trapezoidal stiffener at three different lateral pressures (p) and constant uniaxial compression ($N = 1.974 \times 10^7$ N) in case of two yield stresses (f_Y)

No.	p **[MPa]**	f_Y **[MPa]**	t_F **[mm]**	t_S **[mm]**	$\varphi - 1$ **[-]**
1	0.02	235	23	9	3
2	0.01	235	23	8	3
3	0.005	235	22	8	3
4	0.02	355	17	10	4
5	0.01	355	18	8	5
6	0.005	355	15	8	5

The dimensions with numerical values in Figure 1 are fixed in the calculations, while the other dimensions of stiffeners can be derived from the optimized results of Table 1 based on [13].

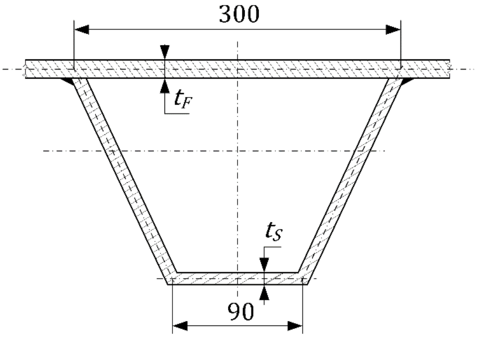

Figure 1
Geometrical designs of the trapezoidal stiffener profiles with the optimized thicknesses

3 Finite Element Analysis

The finite element analysis (FEA) is used to numerically solve differential equations arising in engineering and modelling problems such as vibration analysis problems. The main concept is that the geometry of structures subdivides into non-overlapping smaller parts, called finite elements, which are implemented by the construction of a mesh. The conventional element types possess simply shaped geometry with well-defined stress displacement relationships. If applied loads or geometric nonlinearity were considered so that the best choice is the first-order elements. Thus, the sufficiently refined mesh needs to ensure that the results from simulations are adequate. Concerning shell structures such as our stiffened plate, the Reissner-Mindlin equations are used to model the bending of the plate so that thin and moderately thick plates can be modeled by structural optimization.

According to FE theory, the trapezoidal stiffened plates are meshed into finite elements with fine mesh resolutions, which are four-node reduced integration shell elements (S4R in Abaqus). The approximate global size is specified as 40 mm and yields enough accurate solutions.

3.1 Boundary Conditions

The material is assumed to be isotropic elastic. In our investigation, the stiffened plate is made of steel, provided that the steel grade is characterized by yield stress from Table 1, Young's modulus of $E = 2.1\times10^5$ MPa, Poisson's ratio of $\nu = 0.3$ and density of $\rho = 7.85\times10^{-9}$ t/mm^3.

The uniform thickness plate is stiffened by some trapezoidal-shaped stiffeners and simple support conditions are subjected on all the edges (SSSS) of the base plate. Figure 2 is depicted highlight, the side lengths of the quarter of the base plate parallel to the x-symmetry axis and y-symmetry axis are given by *L/2* and *B/2*, respectively.

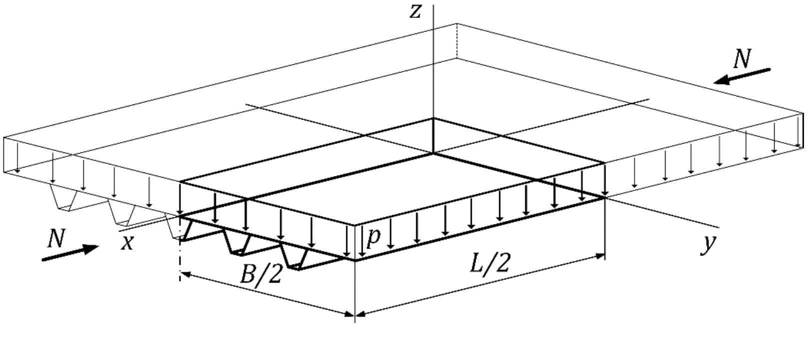

Figure 2
Design of trapezoidal stiffened plate

By exploiting the symmetries of the plate design and loadings, the displacement boundary conditions on the FE model are described on the highlighted quarter structure in Figure 2. The displacement boundary conditions for the points of one-quarter of the stiffened plate are $u_y = \phi_x = \phi_z = 0$ in the z-x plane and in the y-z plane $u_x = \phi_y = \phi_z = 0$ and simply prescribed supports are at the edge of $x = L/2$

according to $u_z = \phi_x = 0$ and at the edge of $y = B/2$ according to $u_z = \phi_y = 0$, where u_x, u_y, and u_z denote the displacements of a point in the mid-plane of the trapezoidal stiffened plate along the x, y, and z directions i.e. longitudinal, transverse and vertical directions, respectively. ϕ_x, ϕ_y, and ϕ_z are the rotations of the normal to the mid-plane at the same point of the structure.

3.2 Modal Analysis and Mode Superposition

To investigate transient vibration analysis for the stiffened plate with trapezoidal stiffeners is performed with these steps, using the commercial software Abaqus. The eigenvalue extraction to calculate the natural frequencies and mode shapes is based on the Lanczos iteration method described in [14], which is used to perform the frequency response analysis or to investigate the eigenvalues for buckling prediction and is applied to extract natural frequencies and modes. Two analysis steps are used for the mode shapes belonging to the trapezoidal stiffened plate subjected to uniaxial compression. The compression, exerted by the force N, is constant at each step. In the first step, the uniaxial compression and geometric nonlinearity are considered so that in the second step, the load stiffness is determined at the end of the first general analysis step and can be included in the eigenvalue extraction.

The rapidly applied pressure on the base plate also causes transient vibrations of the structure, which can be examined by the third step with mode superposition. The first twelve mode shapes in the mode superposition method by the third step are given by creating the second step with static loading.

3.3 Transient Vibration Analysis

Figures 3-5 show the transient responses of the stiffened plates at the center of the base plate, i.e., at the origin of the xyz coordinate system. The damping ratio is considered for the FE models of the welded stiffened plates. Note here that when damping is given as a fraction of critical damping associated with the first sixteen mode shapes, the values used are in the range of 1% to 10% of critical damping [14]. As expected, the peak dynamic deflection values are nearly twice the amount for the critical damping of 1%.

Peak dynamic deflection is a parameter to evaluate deformation upon a sudden applied pressure, and it is calculated as the distance between the front of the system before percussion and its maximum displacement after percussion, which are the absolute values that can be read as the first peak in Figures 3-5. The numerical results of the FE analysis for static and dynamic deflection are summarized in Table 2, which contains the stress values of the peak dynamic deflections at the center of the base plates. Therefore, optimization objective functions [13], such as the central deflections below 1% of length *L,* are also satisfied under dynamic effects, due to sudden, applied pressure.

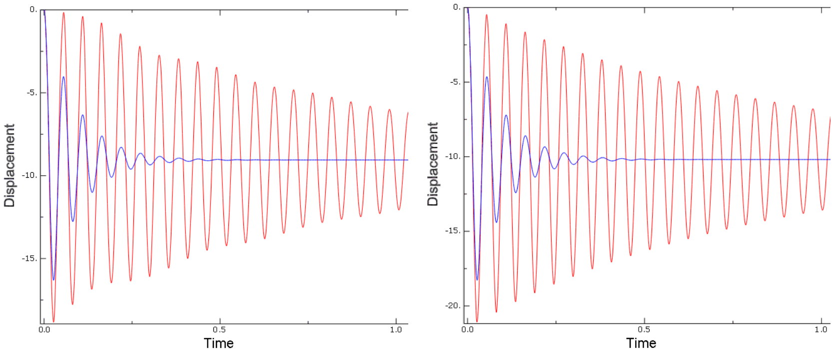

Figure 3

Displacement time-history curve for the critical damping fractions 0.01 (red solid line) and 0.1 (blue solid line) for the magnitude of $p = 0.02$ MPa and yield stresses $f_Y = 235$ MPa and $f_Y = 355$ MPa at the center of the base plate

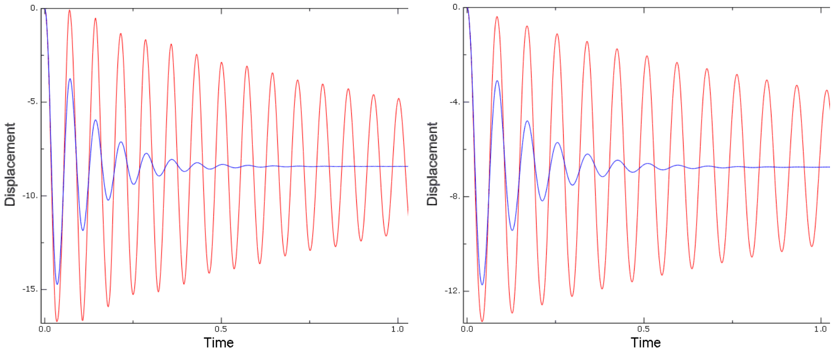

Figure 4

Displacement time-history curve for the critical damping fractions 0.01 (red solid line) and 0.1 (blue solid line) for the magnitude of $p = 0.01$ MPa and yield stresses $f_Y = 235$ MPa and $f_Y = 355$ MPa at the center of the base plate

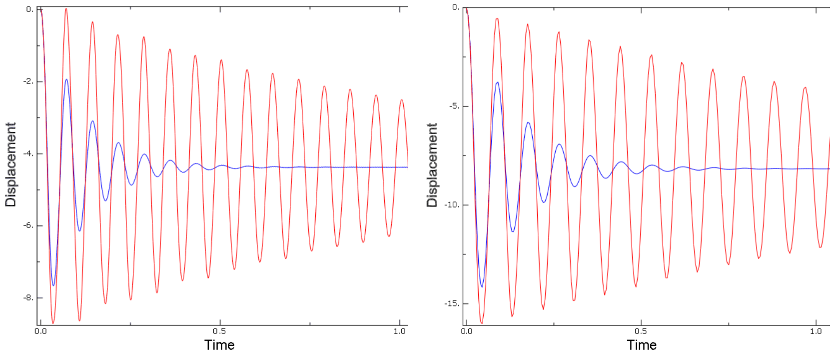

Figure 5

Displacement time-history curve for the critical damping fractions 0.01 (red solid line) and 0.1 (blue solid line) for the magnitude of p = 0.005 MPa and yield stresses f_Y = 235 MPa and f_Y = 355 MPa at the center of the base plate

Table 2

Comparison of peak dynamic and static deflection as well as stress values for dynamic deflection according to the critical damping fraction in the midpoint of the base plate

No.	Static deflection [mm]	Peak dynamic deflection [mm]		stress at midpoint [MPa]	
		$\xi = 0.01$	$\xi = 0.1$	$\xi = 0.01$	$\xi = 0.1$
1	9.05	18.85	16.31	51.71	45.69
2	8.42	16.72	14.74	38.12	33.46
3	4.37	8.72	7.66	20.20	17.72
4	10.2	21.08	18.28	70.79	62.08
5	6.75	13.28	11.73	32.38	28.54
6	8.16	16.02	14.17	42.74	37.56

Due to the structural design, the stresses in Table 2 do not coincide with the maximum value of von Mises stress because the stress concentrations occur at the base of the ribs in the vicinity of the supports. The effect of the optimized ribs and plate thicknesses on the dynamic deflection values is complex, as shown in Table 2, so some finite element analyses are required to clarify the relationships.

Conclusions

The modal superposition method is used in this paper to calculate the transient response of plates accurately and efficiently by applying the shapes modes. The study concludes that the effect of the optimized ribs and plate thicknesses on the dynamic deflection values is complex, so some finite element analyses are required to clarify the relationships. The critical damping fraction is considered for the FE models of the welded stiffened plates the values used commonly used limits of the accepted range.

Other important circumstances are the choice of steel grade which greatly affects the results of dimensions of the optimized stiffened plates and the finite element analysis which predicts the behavior of a structure under a sudden and rapidly applied pressure. This finding highlights the importance of considering material properties when performing forced vibration analysis and underscores the need for careful selection of materials in engineering design. The results of the simulations show that peak dynamic deflections remain below the deflection condition specified in the optimization which is one percent of length L. In general, the plate is suitable for sudden pressure applications, as it has sufficient reserve capacity.

The study highlights the importance of considering material properties in the analysis of forced vibration and emphasizes the need for careful selection of materials in engineering design. From the numerical investigations, it is concluded that FEA can be used safely, to identify the dynamic characteristics of welded stiffened structures. In future work, we will compare the FEA with the experimental results, which can help validate the model results.

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