On Packing of Unequal Squares in a Rectangle

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Abstract: It is known that the sum of the squares of the reciprocals, of positive integer numbers, is finite. It can be asked… What is the smallest area rectangle into which all the squares of sides of length the reciprocals of the positive integers can be packed? In connection with the investigations related to mathability and to applications of computer assisted methods, for considering mathematical problems, an improvement for the best known ϵ is presented, herein. The GNU program, Octave, was used for the calculations.

Keywords: Mathability; Cognitive Infocommunications; Computer Assisted Methods; Packing; Square

1 Introduction

Mathability refers to a branch of cognitive infocommunications that investigates any combination of artificial and natural cognitive capabilities, relevant to mathematics, including a wide spectrum of areas ranging from low-level arithmetic operations, to high-level symbolic reasoning. The concept of Cognitive Infocommunications (CogInfoCom) was introduced in the paper [1]. Some of its further general properties were described in the papers [2] and [3] and in the book [4]. The educational aspects of CogInfoCom and mathability were investigated, among others, in [5-12] while other CogInfoCom related applications of cognitive capabilities are presented in [13-20].

Questions related to mathability and to computer based methods for investigations of mathematical problems have been studied by several authors in recent years [21-25]. T work has contributed to these investigations. A computer assisted method for a packing of squares, of sides of length $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ is presented.

The paper [26], motivated me to present a computer assisted method for a problem of Meir and Moser [27]. The calculation is performed in Octave, available at https://www.gnu.org/software/octave/download.

2 The Problem

It is said the squares $S_1, S_2, S_3, \ldots$ can be packed into a rectangle if it is possible to apply translations and rotations to the sets S_n so that the resulting translated and rotated squares are contained in the rectangle and have mutually disjoint interiors.

Meir and Moser [27], in 1968, originally noted that since:

$$\sum_{i=2}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6} - 1 \tag{1}$$

it is reasonable to ask whether the set of squares of sides of length $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$, which is called the reciprocal squares, can be packed into a rectangle of area $\frac{\pi^2}{6} - 1$. Failing that, find the smallest ϵ such that the reciprocal squares can be packed into a rectangle R of area $\frac{\pi^2}{6} - 1 + \epsilon$. This question can be found in e.g. [28].

Meir and Moser [27] in 1968 showed that the smallest square containing the reciprocal squares is the square of side $\frac{5}{6}$ which shows that $\epsilon < \frac{1}{205}$. Jennings [29] in 1994 gave a rectangle of dimensions $\frac{47}{60} \times \frac{5}{6}$ containing the reciprocal squares which shows that $\epsilon < \frac{1}{127}$

Ball [30] in 1996 gave a rectangle of dimensions $\frac{629}{1\,000} \times \frac{31}{30}$ containing the reciprocal squares which shows that $\epsilon < \frac{1}{198}$

Paulhus [26] in 1997 gave a rectangle of dimensions:

$$0.5 \times \left(2\left(\frac{\pi^2}{6} - 1\right) + 1.606\,553\,066 \cdot 10^{-9}\right) \tag{2}$$

containing the reciprocal squares which shows that:

$$\epsilon < \frac{1}{1\,244\,918\,662} = 8.032\,653\,301 \ldots \cdot 10^{-10} \tag{3}$$

The author [31] in 2018 has found a mistake in the proof of Paulhus. Grzegorek and Januszewski [32] in 2019 filled this gap in the proof of Paulhus. In this paper a new estimate for ϵ is presented.

3 Construction

Theorem 1

The rectangle of dimensions $0.5 \times \left(2\left(\frac{\pi^2}{6} - 1\right) + 1.363\,813\,307\,2 \cdot 10^{-9}\right)$ contains the reciprocal squares which shows that $\epsilon < 6.819\,066\,536 \cdot 10^{-10}$

From the following construction it comes the proof of the theorem.

The square of size $\frac{1}{i}$ is referred by (the square) i. In this paper the width of a rectangle will always refer to the shorter side and the length will always refer to the longer side of the rectangle. Let R be the rectangle of dimensions $\frac{1}{2} \times\ 2\left(\frac{\pi^2}{6}-1\right)$ in which the squares are packed first. It is assumed, that the width of R is horizontal. Let $\mathrm{l}_0 = 0.000\,019\,03$. Let R' be the square of side length l_0.

Let $A = 1\,622\,971\,324$, $B = 1\,648\,721\,271$, $C = 2\,675\,827\,341$, $D = 2\,718\,281\,828$, $E = 2\,761\,408\,695$. The numbers A, B, C, D and E comes from [26]. Let $n_0 = E + 1$, $n_i = \lfloor n_{i-1}(1 + l_0)\rfloor$ for $i \geq\ 1$ where $\lfloor . \rfloor$ is the floor function and $n'_{18} = 2\,762\,386\,911$. Observe the squares from n_{i-1} to $n_i - 1$ fit in a row of R'. Let $A_1 = 1\,016\,225\,800$, $A_2 = 1\,000\,000\,440$, $C' = 2\,633\,103\,139$ and $C_0 = 2\,674\,879\,766$, ..., $C_{18} = 2\,675\,796\,170$ so that

$$\sum_{i=C_j}^{C_{j+1}-1} \frac{1}{i} < l_0 \leq \sum_{i=C_j}^{C_{j+1}} \frac{1}{i} \quad \text{for } \mathrm{j} = 0, \dots, 18 \tag{4}$$

The numbers $C_1, \dots, C_{18}$ are calculated with the help of Octave and Lemma 1.

Lemma 1. The following is true:

$$\ln\frac{n+m+1}{n} < \frac{1}{n} + \cdots + \frac{1}{n+m} < \ln\frac{n+m}{n-1} \tag{5}$$

where n and m are positive integers and $n \neq 1$

Proof of Lemma 1. After using the lower and upper sums of the function

$x \mapsto \frac{1}{x}$ the estimates are trivial.

By Lemma 1, it is true:

$$\frac{1}{n} + \cdots + \frac{1}{n+m} \approx \frac{\ln\frac{n+m+1}{n} + \ln\frac{n+m}{n-1}}{2} \tag{6}$$

The numerical estimations based on the Lemma 1. The following two short Octave retval functions help the calculations:

```
function retval = distU (k,v)
  retval = log(v/(k-1));
endfunction
function retval = distL (k,v)
  retval = log((v+1)/k);
endfunction.
```

By Lemma 1, the function distL(k,v) returns an lower bound of the sum $\frac{1}{k}+\cdots+\frac{1}{v}$ and the function distU(k,v) returns an upper bound of the sum $\frac{1}{k}+\cdots+\frac{1}{v}$.

The following is used:

Lemma 2. The squares C_0, C_0+1, ..., C, n'_{18}, $n'_{18}+1$, ..., $n_{100001}-1$, $n_{100\,251}$, $n_{100\,251}+1$, ... can be packed in R'.

Proof of Lemma 2. The squares from C_0 to C are packed in rows of length no greater than l_0 in the square R'.

If $1\leq i\leq 18$, then in the ith row the squares go from C_{i-1} to C_i-1 (Figure 1). Observe, the squares from C_{i-1} to C_{i-1} fit in the i^{th} row.

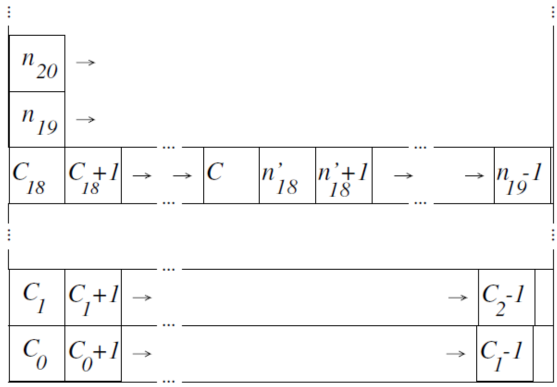

Figure 1
The ith row in R' ($1\leq i\leq 19$)

By Lemma 1 and Octave, in the 19th row the squares go from C_{18} to C and from n'_{18} to $n_{19}-1$. If $20\leq i\leq 100\,001$, then in the ith row the squares go from n_{i-1} to n_i-1. Observe, the squares from n_{i-1} to n_i-1 fit in the i^{th} row. If $100\,002\leq i$, then in the i^{th} row the squares go from n_{i+249} to $n_{i+250}-1$ (Figure 2). Observe, the squares from n_{i+249} to $n_{i+250}-1$ fit in the i^{th} row.

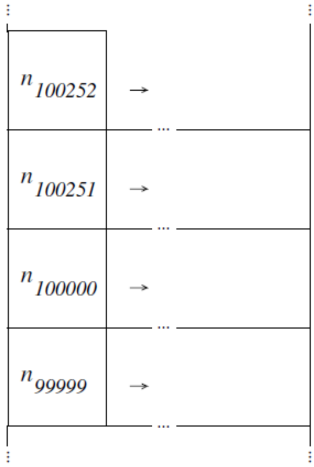

Figure 2
The ith row in R' ($100\,000 \leq\ i \leq 100\,003$)

Now:

$$\begin{aligned} n_i &= \lfloor n_{i-1}(1+l_0)\rfloor > n_{i-1}\left(1+l_0-\frac{1}{n_{i-1}}\right) \\ &> n_{i-2}\left(1+l_0-\frac{1}{n_{i-1}}\right)\left(1+l_0-\frac{1}{n_{i-2}}\right) \\ &> n_{i-2}\left(1+l_0-\frac{1}{n_{i-2}}\right)^2 > \cdots > n_{18}\left(1+l_0-\frac{1}{n_{18}}\right)^{i-18} \end{aligned} \tag{7}$$

for $i > 18$, Thus:

$$\sum_{i=1}^{19}\frac{1}{C_{i-1}}+\sum_{i=20}^{100\,001}\frac{1}{n_{i-1}}+\sum_{i=100\,002}^{\infty}\frac{1}{n_{i+249}}$$

$$< \ 0.000\,000\,007\,101\,90\ldots+\sum_{i=20}^{100\,001}\frac{1}{n_{18}\left(1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}\right)^{i-19}}$$

$$\ldots+\sum_{i=100\,002}^{\infty}\frac{1}{n_{18}\left(1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}\right)^{i+231}}$$

$$= \ 0.000\,000\,007\,101\,90\ldots+\frac{1}{n_{18}\left(1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}\right)}\sum_{i=0}^{99\,981}\frac{1}{\left(1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}\right)^{i}}$$

$$
+\frac{1}{n_{18}\left(1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}\right)^{100\,233}}\sum_{i=0}^{\infty}\frac{1}{\left(1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}\right)^{i}}
$$

$$
=\ 0.000\,000\,007\,101\,90\,\ldots+\frac{1}{n_{18}\left(1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}\right)}\frac{1-\frac{1}{\left(1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}\right)^{99\,982}}}{1-\frac{1}{1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}}}
$$

$$
+\frac{1}{n_{18}\left(1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}\right)^{100\,233}}\frac{1}{1-\frac{1}{1+\mathrm{l}_0-\frac{1}{\mathrm{n}_{18}}}}<0.000\,019\,017<l_0 \tag{8}
$$

Octave was used to the numerical calculations. Thus the squares fit in R', which is the statement of the lemma.

Proof of Theorem 1. It is assumed that the squares up to 10^9 are packed in R as in [26]. By [26], there is a rectangle R_L of length and width at most l_0, which has no common interior point with the squares up to 10^9. By Lemma 2, it is necessary to find a place to pack the squares from 10^9+1 to C_0-1, from $C+1$ to $n'_{18}-1$ and from $n_{100\,001}$ to $n_{100\,251}-1$.

Let the squares from 10^9+1 to C_0-1 and from $C+1$ to $n'_{18}-1$ be packed into a rectangle R_N of length $\frac{1}{2}$ as in Figure 3. By [26], the squares from 10^9+1 to B and from $C+1$ to D can be arranged as in Figure 3. By Lemma 1, $\frac{1}{C'}+\cdots+\frac{1}{C_0-1}<\frac{1}{A}+\cdots+\frac{1}{B}$.

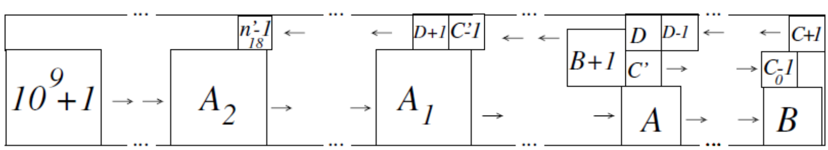

Figure 3

The squares in R_N

It is shown that the highest horizontal edge belongs to the square $C'-1$ thus, the width of the rectangle R_N is $\frac{1}{A_1}+\frac{1}{C'-1}=\ 1.363\,813\,307\,18\,\ldots\cdot 10^{-9}$.

First, it is shown that the highest horizontal edge belongs to the square $C'-1$ among the squares $B+1$, ..., $C'-1$. The square $C'-y$ *sits* on the square A_1+x if the relative interior of the bottom side of $C'-y$ and the relative interior of the upper side of A_1+x have a nonempty intersection. Since:

$$\frac{\frac{1}{A_1}}{\frac{1}{C'-1}} = 2.5 \ldots \tag{9}$$

at most three squares sit on the square $A_1 + x$ if x is a (small) positive integer. It is assumed, that the square $C' - y$ sits on the square $A_1 + x$. Thus

$$\frac{1}{A_1} + \frac{1}{C'-1} > \frac{1}{A_1+x} + \frac{1}{C'-1-3x} \geq \frac{1}{A_1+x} \frac{1}{C'-y} \tag{10}$$

if $0 < x \leq\ x_1$, where $x_1 = 350\ 300\ 705$ (the value of x_1 comes from the Octave). By Lemma 1, the square $y_1 = 1\ 958\ 123\ 269$ sits on the square $A_1 + x_1$, but $y_1 + 1$ does not sit on $A_1 + x_1$. Since:

$$\frac{\frac{1}{A_1+x}}{\frac{1}{y_1}} = 1.4 \ldots \tag{11}$$

at most two squares sit on the square $A_1 + x_1 + x$ if x is a (small) positive integer. It is assumed, that the square $y_1 - y$ sits on $A_1 + x_1 + x$. Thus:

$$\frac{1}{A_1} + \frac{1}{C'-1} > \frac{1}{A_1+x_1+x} + \frac{1}{y_1-2x} \tag{12}$$

$$\geq \frac{1}{A_1+x_1+x} + \frac{1}{y_1-y} \tag{13}$$

if $0 < x \leq x_2$, where $x_2 = 334\ 746\ 954$ (the value of x_2 comes from the Octave). Since $A_1 + x_1 + x_2 > A$, the highest horizontal edge belongs to the square $C' - 1$ among the squares $B + 1$, ..., $C' - 1$.

Similarly:

$$\frac{1}{A_2} + \frac{1}{n'_{18}-1} > \frac{1}{A_2+x} + \frac{1}{n'_{18}-1-3x} \tag{14}$$

if $0 < x \leq\ A_2 - A_1 - 1$

and

$$\frac{1}{C'}+\frac{1}{D'-1}>\frac{1}{C'+x}+\frac{1}{D-2x} \tag{15}$$

if $0<x\leq\ C_0-C'-2$

The candidates of the width of the rectangle R_N are

$$\frac{1}{A_2}+\frac{1}{n'_{18}-1}=1.362\ 005\ 3\ \ldots\cdot 10^{-9}$$

$$\frac{1}{A_1}+\frac{1}{C'-1}=1.363\ 813\ 307\ 19\ \ldots\cdot 10^{-9}$$

$$\frac{1}{A}+\frac{1}{C'}\ +\frac{1}{D-1}=1.363\ 813\ 307\ 18\ \ldots\cdot 10^{-9} \tag{16}$$

Thus the width of the rectangle R_N is $\frac{1}{A_1}+\frac{1}{C'-1}=1.363\ 813\ 307\ 19\ \ldots\cdot 10^{-9}$

It is necessary to find a place to pack the squares $n_{100\ 001}$ to $n_{100\ 251}-1$.

Let:

$$b_1=\frac{1}{n_{18}\left(1+l_0-\frac{1}{n_{18}}\right)^{99\ 983}}\ =5.400\ 354\ 04\ \ldots\cdot\ 10^{-11} \tag{17}$$

Now:

$$\frac{1}{n_i}\leq\frac{1}{n_{100\ 001}}<\frac{1}{n_{18}\left(1+l_0-\frac{1}{n_{18}}\right)^{99\ 983}}=b_1 \tag{18}$$

for $i=100\ 001,\ldots,100\ 250$ and let:

$$b_2=\frac{1}{n_{100\ 001}}+\frac{1}{n_{100\ 001}+1}+\cdots+\frac{1}{n_{100\ 251}-1}<250\cdot\ l_0 \tag{19}$$

The square $y_n=1\ 656\ 583\ 751$ sits on the square $x_n=1\ 615\ 268\ 375$. (Observe, y_n-1 does not sit on x_n.) Thus:

$$\sum_{\mathrm{i=B+1}}^{\mathrm{y_n-1}}\frac{1}{i}\ <250\cdot\ \mathrm{l_0}<\sum_{\mathrm{i=B+1}}^{\mathrm{y_n}}\frac{1}{i} \tag{20}$$

and

$$\sum_{\mathrm{i=x_n+1}}^{\mathrm{A-1}} \frac{1}{i} < 250 \cdot \mathrm{l_0} < \sum_{\mathrm{i=x_n}}^{\mathrm{A-1}} \frac{1}{i} \tag{21}$$

Let:

$$h = \frac{1}{A_1} + \frac{1}{C' - 1} - \frac{1}{x_n} - \frac{1}{y_n} = 1.410\,719\,974\,85 \ldots \cdot 10^{-10} \tag{22}$$

Since the highest horizontal edge belongs to the square y_n among the squares $B + 1$, ..., y_n, there is a rectangle R_f of dimensions $250 \cdot l_0 \times h$ which has no common interior point with the squares up to $n'_{18} - 1$. Since $b_1 < h$ and $b_2 < 250 \cdot l_0$, the squares from $n_{100\,001}$ to $n_{100\,251} - 1$ fit in R_f.

Thus the reciprocal squares are contained in a rectangle of dimensions

$$\frac{1}{2} \times \left(2\left(\frac{\pi^2}{6} - 1\right) + 1.363\,813\,307\,19 \ldots \cdot 10^{-9}\right) \tag{23}$$

which shows that $\epsilon \leq 6.819\,066\,535\,97 \ldots \cdot 10^{-10}$

Conclusions

From the above proof, it should be recognised, that performing difficult calculations with the help of computers and/or suitable programs, can be an easy task. Without a computer, the calculations on a piece of paper, indeed, take a very long time.

Calculations with Octave and the two short retval functions, is an easy task. The numbers A, B, C, D and E come from [26]. The numbers $A_1, A_2, C', C_0, C_1, \ldots, C_{18}$ and the width of the rectangle R_N are calculated with the help of Octave. By Lemma 1, the control of round-off errors is achieved.

The packing question in this paper, for ϵ, was asked back in 1968, and the question is still open. In the papers [26] [27] [29] [30] improved estimates for the value of ϵ can be found, but these estimates were not final, as you can see by the evidence of this paper. This short work should inspire authors to closely examine long standing mathematical questions, with the help of a computer.

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