

# Analytical Identification of Critical Section of Axle of Freight Wagon

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*Abstract: The paper deals with the analytical identification of the critical section of the axle of a freight wagon. The procedure for calculating the strength of railway axles in accordance with the European standard EN 13103 is shown. Based on this, a method for identifying the critical section of the axle was presented. The goal is to find the section or zone with the lowest dynamic safety factor. The proposed method was applied to a specific example of standard axle of freight wagon for an axle load of 22.5 tons. The obtained results have shown that the critical section is located in the zone of transition radius between the wheel-seat and the middle part of the axle. The obtained analytical results were verified by FEM calculation in the ANSYS software package. The results of the research presented in the paper may be interesting and should be considered for future design, optimization and standardization of axles of freight wagons.*

*Keywords: railway axle; freight wagon; critical section; EN 13103*

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## 1 Introduction

The axles of railway vehicles are among the most responsible elements in the entire railway system. The quality of functioning and reliability of railway axles directly affects the safety and security on the railway. Failure of the railway axle in running very often leads to the derailment of a given railway vehicle and sometimes of a larger part or the whole train (one example is shown in Fig. 1) [1]. The consequences of these events are great direct and indirect material damage, and very often human casualties. Therefore, research on the problems of railway axles takes the one of the most important places in the railway engineering [2, 3].

Nowadays, there are many developed methods for analyzing and investigation the behavior of vital elements of railway vehicles and track [4-11]. Actual research on railway axles is primarily related to analyzing their behavior and characteristics as extremely dynamically loaded elements. Martínez-Casas et al. have investigated the numerical determination of the stresses of railway axles using the model of

interaction between the train and track [12]. In [13] and [14] Nikolov and Krishna Sudha *et al.* have studied the strength characteristics and design of locomotive axles. Xue *et al.* have analyzed the problem of assessing the reliability of axle of a freight wagon [15], while Dikmen *et al.* and Yasniy *et al.* have investigated the problem of estimating the lifetime of the railway axle [16, 17]. Given that railway axles belong to unsprung masses, the problem of reducing their weight is a constantly current topic. In [18] Han *et al.* have researched how to reduce the weight of axle of a city railway vehicle. Son *et al.* have studied the problems of designing a hollow railway axle [19]. In order to reduce unsprung masses, there are approaches to introduce new materials of wheels and axles. In this regard, Bruni *et al.* and Mistry *et al.* have studied the design of a railway axle made of composite material [20, 21].



Figure 1

Example of derailment of freight train caused by axle failure [1]

The aim of this paper is to establish an analytical method for identifying the critical section of the axle of a freight wagon. The main goal is to identify the cross-section of the axle with the lowest dynamic safety factor. In doing so, all relevant loads for axle calculation and its strength requirements are taken from the relevant international standard EN 13103 [22]. Lastly, the obtained analytical

results should be verified by numerical calculations using FEM, which are successfully used for a long time in the research of phenomena related to wheel and rail [23, 24].

## 2 Strength Calculation in Accordance with EN 13103

The European standard EN 13103 prescribes authoritative loads for the strength calculation of axles of railway vehicles. The scheme of loads for calculation is given in Fig. 2. The forces of action on the axle journals are [22]:

$$P_1 = \left( 0.625 + 0.075 \frac{h_o}{b} \right) m_1 \cdot g \quad (1)$$

$$P_2 = \left( 0.625 - 0.075 \frac{h_o}{b} \right) m_1 \cdot g \quad (2)$$

$$H \approx Y_1 - Y_2 \quad (3)$$

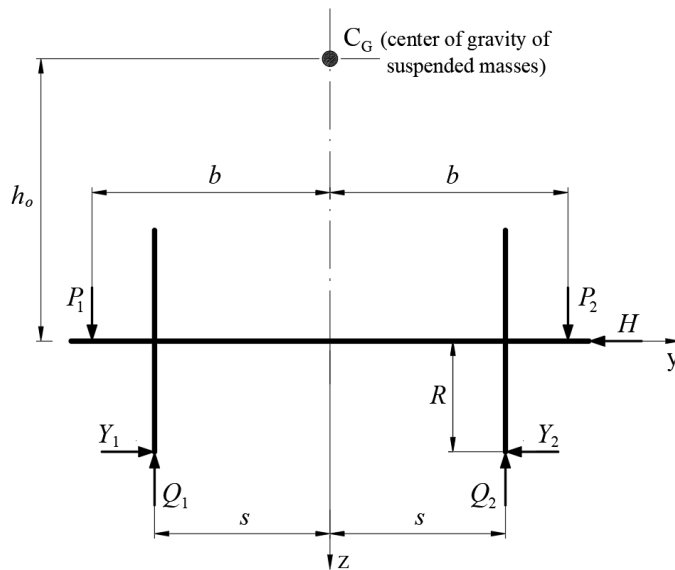


Figure 2

Scheme of loads for strength calculation of railway axles [22]

The reactive forces in the wheel-rail contacts are [22]:

$$Y_1 = 0.3 \cdot m_1 \cdot g \quad (4)$$

$$Y_2 = 0.15 \cdot m_1 \cdot g \quad (5)$$

$$Q_1 = \frac{1}{2s} [P_1(b+s) - P_2(b-s) + (Y_1 - Y_2) \cdot R] \quad (6)$$

$$Q_2 = \frac{1}{2s} [P_2(b+s) - P_1(b-s) - (Y_1 - Y_2) \cdot R] \quad (7)$$

In the previous expressions  $h_o$  is height of gravity center of suspended masses,  $b$  is half-distance between axle-box cases,  $s$  is half-distance between nominal rolling circles,  $R$  is wheel radius, and  $m_1$  is mass, which is defined as the difference between axle-load mass ( $m_{al}$ ) and wheelset mass ( $m_{ws}$ ):

$$m_1 = m_{al} - m_{ws} \quad (8)$$

The diagram of bending moments of the axle in the  $yz$  plane is given in Fig. 3.

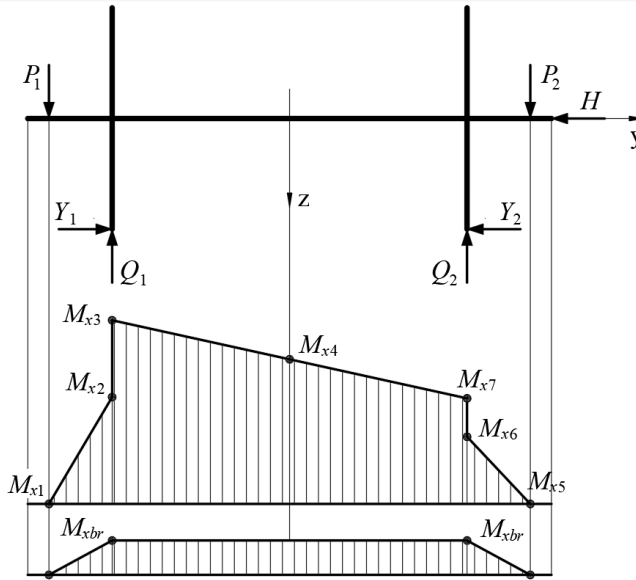


Figure 3

Diagram of bending moments of axle in  $yz$  plane

The bending moments in the plane  $yz$ , caused by the loads of axle defined in Fig. 1 are defined by the following expressions:

$$M_{x1} = 0 \quad (9)$$

$$M_{x2} = P_1(b-s) \quad (10)$$

$$M_{x3} = P_1(b-s) + Y_1 \cdot R \quad (11)$$

$$M_{x4} = P_1 \cdot b + Y_1 \cdot R - Q_1 \cdot s \quad (12)$$

$$M_{x5} = 0 \quad (13)$$

$$M_{x6} = P_2 (b - s) \quad (14)$$

$$M_{x7} = P_2 (b - s) + Y_2 \cdot R \quad (15)$$

The maximum bending moment in the plane yz, caused by the braking with shoe brake is defined by the following expression:

$$M_{xbr} = \mu \cdot 0.3 \cdot F_s (b - s) \quad (16)$$

where:  $\mu$  – friction coefficient between shoe and wheel,  $F_s = B_k \cdot P_w$  – braking force,  $B_k$  – braking coefficient,  $P_w = (m_1 + m_{ws}) \cdot g / 2$  – wheel static load.

The maximum bending moment in the plane xy (Fig. 4), caused by the braking is defined by the following expression:

$$M_{zbr} = 0.3 \cdot F_s (b - s) \quad (17)$$

The torsion moment of the axle (Fig. 4), caused by the braking is defined by the following expression:

$$M_{ybr} = 0.3 \cdot P_w \cdot R \quad (18)$$

In any axle cross-section, the equivalent stress  $\sigma_e$  must be less than or equal to permissible stress  $\sigma_{per}$ :

$$\sigma_e \leq \sigma_{per} \quad (19)$$

The equivalent stress is determined by the following expression:

$$\sigma_e = S_k \cdot \frac{32M_e}{\pi d^3} \quad (20)$$

where:

$$M_e = \sqrt{M_x^2 + M_y^2 + M_z^2} \text{ – equivalent moment in considered cross-section}$$

$d$  – axle diameter in considered cross-section

$S_k$  – stress concentration factor in considered cross-section

The stress concentration factor depends on the location of the considered cross-section and is defined by the expressions:

$$S_k = \frac{(4-Y)(Y-1)}{5 \cdot (10X)^{(2.5X+1.5-0.5Y)} + 1}; \quad X = \frac{r}{D}; \quad Y = \frac{D}{d} \quad (21)$$

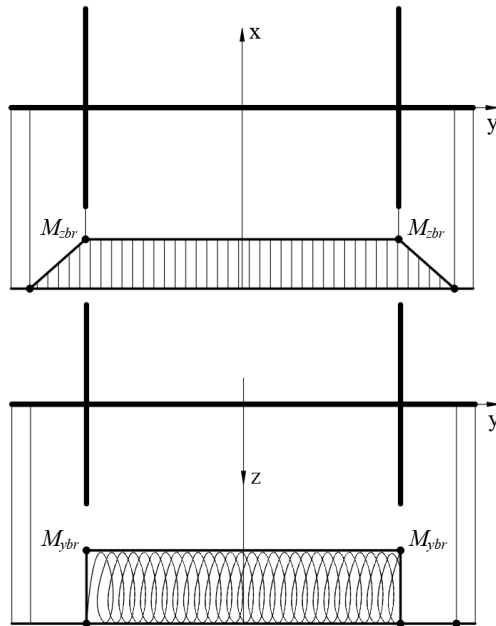


Figure 4

Diagram of bending moment of axle in xy plane and diagram of torsion moment of axle

The explication of parameters  $d$ ,  $D$  and  $r$  in expression (21) is given in Fig. 5.

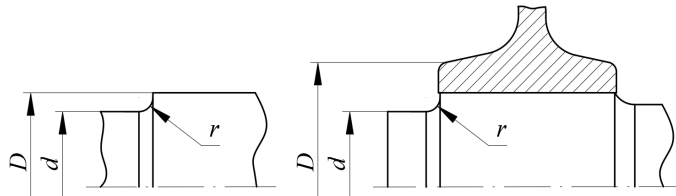


Figure 5

Explication of parameters  $d$ ,  $D$  and  $r$

The permissible stress is defined as the ratio between the endurance limit  $\sigma_N$  for a given axle material, and the safety factor  $s$ :

$$\sigma_{per} = \frac{\sigma_N}{s} \tag{22}$$

### 3 Analytical Identification of Critical Section

The subject of research is the standard axle of a freight wagon for an axle load of 22.5 tons, shown in Fig. 6. In order to perform analytical identification of critical section of the considered axle, five representative cross-sections S1-S5 for are selected for strength calculation. It is important to note that these cross-sections are selected in the places where pressed joints or transition radii between different diameters of the axle are present.

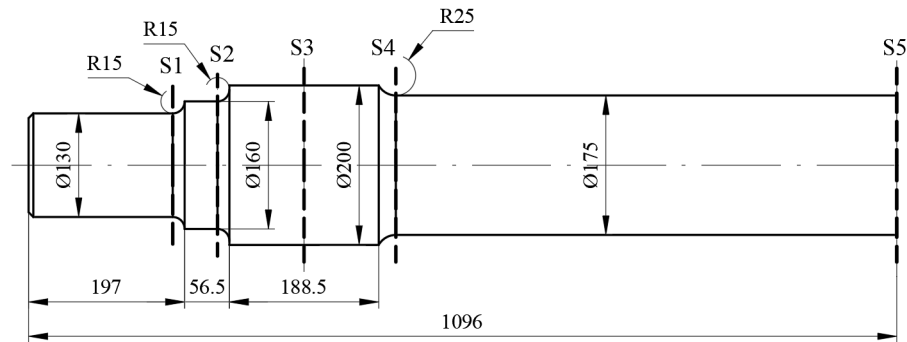


Figure 6

Subject of research – standard axle of a freight wagon for an axle load of 22.5 tons

The input parameters for the calculation of the strength of the considered axle are given in the Table 1.

Table 1  
Values of input parameters for calculation of strength of considered axle

$m_{al} = 22500$ kg	axle-load
$m_{ws} = 1074$ kg	wheelset mass
Axle material	EA1T
$h_o = 1900$ mm	height of center of gravity of suspended mass for loaded wagon
$2b = 2000$ mm	distance between axle-box cases
$2s = 1500$ mm	distance between nominal rolling circles
$D = 920$ mm	nominal wheel diameter

The obtained values of parameters for the determination of axle stresses, obtained on the basis of expressions given in the Chapter 2, are given in the Table 2.

The expressions for the determination of equivalent moments in considered cross-sections of the axle are:

$$M_{eS1} = \sqrt{\left(\frac{0.086 \cdot M_{x2} + 0.086 \cdot M_{xbr}}{0.252}\right)^2 + \left(\frac{0.086 \cdot M_{zbr}}{0.252}\right)^2} \quad (23)$$

$$M_{eS2} = \sqrt{\left(\frac{0.1425 \cdot M_{x2} + 0.1425 \cdot M_{xbr}}{0.252}\right)^2 + \left(\frac{0.1425 \cdot M_{zbr}}{0.252}\right)^2} \quad (24)$$

$$M_{eS3} = \sqrt{(M_{x3} + M_{xbr})^2 + M_{ybr}^2 + M_{zbr}^2} \quad (25)$$

$$M_{eS4} = \sqrt{\left(\frac{0.632 \cdot (M_{x3} - M_{x4})}{0.75} + M_{x4} + M_{xbr}\right)^2 + M_{ybr}^2 + M_{zbr}^2} \quad (26)$$

$$M_{eS5} = \sqrt{(M_{x4} + M_{xbr})^2 + M_{ybr}^2 + M_{zbr}^2} \quad (27)$$

Table 2

Obtained values of parameters for determination of axle stresses, obtained on basis of expressions given in Chapter 2

Parameter	Value	Parameter	Value	Parameter	Value
$m_1$ [kg]	21426	$M_{x1}$ [kNcm]	0	$B_k$	0.75
$P_1$ [kN]	161.32	$M_{x2}$ [kNcm]	4033	$F_s$ [kN]	82.77
$P_2$ [kN]	101.42	$M_{x3}$ [kNcm]	6933.76	$\mu$	0.1
$H$ [kN]	31.53	$M_{x4}$ [kNcm]	4009.63	$M_{xbr}$ [kNcm]	62.08
$Y_1$ [kN]	63.06	$M_{x5}$ [kNcm]	0	$M_{zbr}$ [kNcm]	620.78
$Y_2$ [kN]	31.53	$M_{x6}$ [kNcm]	2535.5	$M_{ybr}$ [kNcm]	1522.97
$Q_1$ [kN]	180.97	$M_{x7}$ [kNcm]	3985.88		
$Q_2$ [kN]	81.77	$P_w$ [kN]	110.36		

The obtained values of safety factors and equivalent moments in the considered axle cross-sections are given in the Table 3.

Table 3

Obtained values of safety factors and equivalent moments in considered axle cross-sections

Cross-section	Stress concentration factor $S_k$	Equivalent moment $M_e$ [kNcm]
S1	1.11	1413.49
S2	1.36	2342.13
S3	1.00	7186.56
S4	1.19	6739.52
S5	1.00	4391.32

Finally, the obtained values of equivalent stresses in considered axle cross-sections are given in the Table 4. The table also gives the values of endurance limits defined by [22] in considered axle cross-sections and safety factor as a ratio between endurance limit and equivalent stress.

The obtained results have shown that the most critical section of the considered axle is located in the zone of transition radius between the wheel-seat and the middle part of the axle (zone between cross-section S3 and S4 shown in Fig. 6).



Table 4

Obtained values of equivalent stresses, endurance limits and safety factors in considered axle cross-sections

Cross-section	Equivalent stress	Endurance limit	Safety factor
	$\sigma_e$ [kN/cm <sup>2</sup> ]	$\sigma_N$ [kN/cm <sup>2</sup> ]	$s$
S1	7.28	12	1.65
S2	7.93	12	1.51
S3	9.15	12	1.31
S4	15.25	20	1.31
S5	8.35	20	2.39

This result is in agreement with many cases of failures of railway axles that have occurred in the past, where the axles have fractured right in the identified zone [1, 25-27].

## 4 Verification of Analytical Results

The obtained analytical results were verified by FEM (finite element method) calculation in the ANSYS software package. The formed numerical model consists of the considered axle and a pair of wheels. The model is composed of 140359 finite elements and 233962 nodes (Fig. 7).

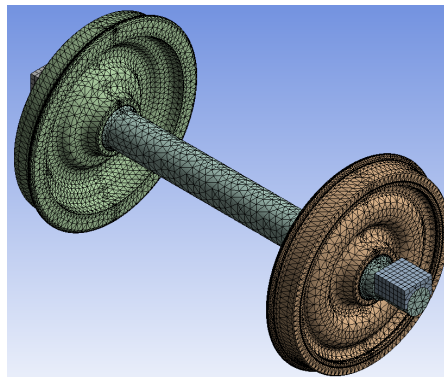


Figure 7

Numerical FEM model (140359 finite elements and 233962 nodes)

The obtained numerical results for the equivalent stresses of the axle in the considered cross-sections from Chapter 3 are shown in Fig. 8.

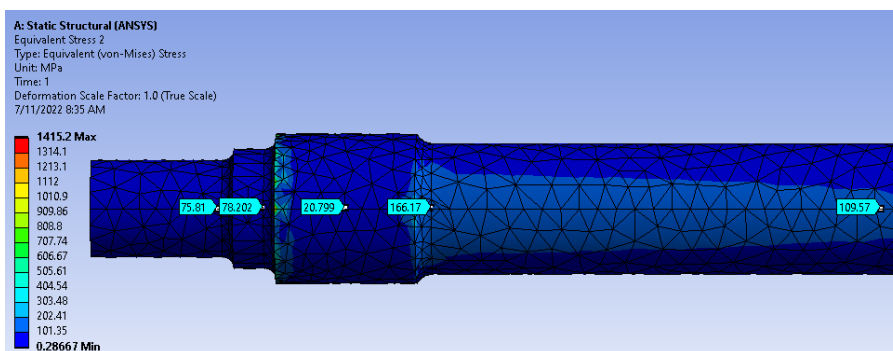


Figure 8

Obtained numerical results for equivalent stresses in considered cross-sections of axle

A comparative view of the equivalent stresses obtained by analytical and numerical way is given in Table 5.

Table 5

Comparative view of equivalent stresses obtained by analytical and numerical way

Cross-section	Analytical results $\sigma_e$ [kN/cm <sup>2</sup> ]	FEM results $\sigma_e$ [kN/cm <sup>2</sup> ]
S1	7.28	7.58
S2	7.93	7.82
S3	9.15	2.08
S4	15.25	16.6
S5	8.35	10.96

The obtained FEM results have shown good degree of agreement with the analytical results. There is only one exception in section S3, where the contact surface with the wheel is present in the numerical model. Therefore, a lower value of equivalent stress is obtained in this section. In general, it can be concluded that the analytical and numerical results are in agreement and that the analytically identified critical zone of the axle in chapter 3 is valid.

## Conclusions

The paper presents an analytical method for identifying the critical section of axle of a freight wagon. The method is based on the requirements of the European standard EN 13103 for axle strength. Analytical results for standard axle of freight wagons for an axle-load of 22.5 t have shown that the critical section is located in the zone of transition radius between the wheel-seat and the middle part of the axle. In this zone, the axle has the lowest safety factor of about 1.3, expressed as the ratio between the endurance limit and the equivalent stress. This result is in correspondence with many cases of failures of axles from exploitation that have broken just in the identified zone. The established analytical procedure was

verified by the results of FEM calculation, while the analytical and numerical results are in correspondence. The outcomes of the paper should be taken into account in the further design, optimization and standardization of axles of freight wagons.

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## Nomenclature

Symbol	Meaning
$P_1, P_2$ [kN]	vertical forces on the axle journals
$H$ [kN]	lateral force on the axle journal
$Q_1, Q_2$ [kN]	vertical forces in the wheel-rail contacts
$Y_1, Y_2$ [kN]	lateral forces in the wheel-rail contacts
$h_o$ [cm]	height of gravity center of suspended masses
$b$ [cm]	half-distance between axle-box cases
$s$ [cm]	half-distance between nominal rolling circle
$R$ [cm]	wheel radius
$m_{al}$ [kg]	axle-load mass
$m_{ws}$ [kg]	wheelset mass
$M_x$ [kNcm]	bending moment in the plane yz
$M_{xbr}$ [kNcm]	bending moment in the plane yz caused by the braking

$\mu$	friction coefficient between shoe and wheel
$F_s$ [kN]	braking force
$B_k$	braking coefficient
$P_w$ [kN]	wheel static load
$M_{zbr}$ [kNcm]	bending moment in the plane xy caused by the braking
$M_{ybr}$ [kNcm]	torsion moment of the axle caused by the braking
$\sigma_e$ [kN/cm <sup>2</sup> ]	equivalent stress
$\sigma_{per}$ [kN/cm <sup>2</sup> ]	permissible stress
$M_e$ [kNcm]	equivalent moment
$d$ [cm]	axle diameter in considered cross-section
$S_k$	stress concentration factor
$\sigma_N$ [kN/cm <sup>2</sup> ]	endurance limit
$s$	safety factor