A Comparative, Experimental Study of Full Factorial and Central Composite Designs, Through Machinability Analysis of Aluminum Alloy

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Abstract—**Machinability experiments are usually designed using different Design Of Experiment (DOE) methods. The different designs require disparate period of time and cost. Furthermore, the gained information is also different. The primary aim of the present study was a comparative analysis of different designs of experiments. A full factorial (FF) and a Central Composite Inscribed (CCI) design were used in order to analyze the machinability of a 6082 aluminum alloy. The feed rate and the cutting speed were chosen as factors in order to analyze their influence on the cutting force. The main effects and the interactions of the factors were analyzed with the analysis of variance (ANOVA) technique. The collected information, experiment cost and time were compared and discussed in order to realize the differences between the full factorial and the fractional factorial design. It was found that the ratio of the cutting force estimated through the CCI design to the cutting force estimated through the FF design was less than 1.13 all over the factor space.**

I. Introduction

The efficiency, cost reduction, and the sustainability became more essential aspects today, which are justified by customer needs and environmental awareness. The realization of these needs can be done in a lot of different ways during the design process, among which the structure- and process optimization methods can be highlighted [1, 2]. The optimization methods are based mainly on analytical models or experiments. The Design Of Experiment (DOE) is an effective technique to define factors and their levels, response variables (optimization parameters) and other important data in order to minimize the experimental settings and maximize the collected information [1, 3-7]. The schematic figure of the research object can be seen in the Fig. 1.

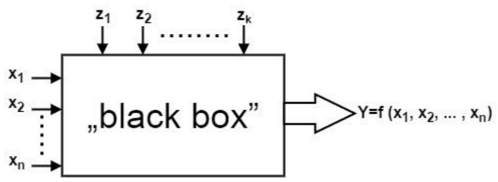

Figure 1. Schematic figure of the research object: the "black box"

Fig. 1 shows that the factors (x_i) and the noises (z_i) are the input parameters, and the optimization parameter (Y) is the response variable of the analyzed process (which is inside the "black box"). The $Y=f(x_i)$ function is searched in order to describe the analyzed process with the help of the factors.

DOE offers a large number of different planning methods (ex. Full Factorial, Central Composite, Box-Wilson, Box-Behnken, Taguchi, Plackett-Burman etc.), which can be used for experimental settings with different quantities and layouts [3, 4]. The visualization of some of the above mentioned DOE methods are illustrated in Fig. 2. The results of experiments are affected by these settings [6].

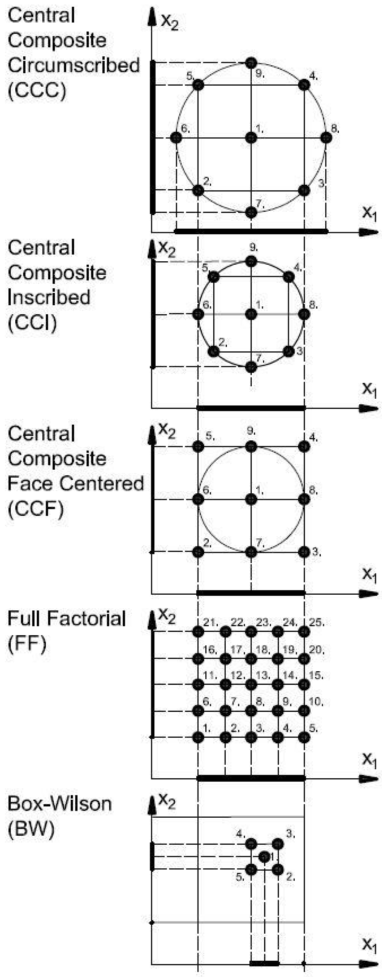

Figure 2. Schematic figure of different DOE methods in case of two factors (x_1 and x_2)

The main objective of our research is to design machining experiments based on full factorial (FF) and central composite inscribed (CCI) DOE methods, and then to compare their information content. We analyzed that in comparison with the full factorial experiments, how different will be the obtained information with the CCI method during the machinability analysis.

The evaluation of the CCI results was performed by the response surface methodology (RSM), the response variables (optimization parameters) were searched in second-order polynomial form, due to the analyzed machining process is non-linear [8, 9]. The general form of the second-order polynomial is described by Eq. (1)

$$Y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ii} x_{ii}^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij} x_i x_j + \delta \qquad (1)$$

, where x_i are factors, $x_i x_j$ are the interactions of those factors, b_0 is a constant, b_i, b_{ii}, b_{ij} are regression coefficients, δ is an accidental error, and Y is the response variable.

The main effects and interactions have been analyzed by the Analysis of Variance (ANOVA) statistical technique. The obtained information content, and the time and cost expenditures have been compared, and the consequences of the experiment were discussed.

II. Experimental Settings

The response variable (Y - optimization parameter) analyzed in this study is the cutting force and this components, when face milling a 6028 aluminum alloy with geometry of 60 x 60 x 100mm.

The experiments were carried out on a three-axis Kondia B640 machining center with NCT100 control. A diameter of 21 mm MITSUBISHI AQXR212SA20S end mill was used. The cutting force was detected with a KISTLER 9257 three-component load cell at a sampling frequency of 8 kHz, while the data was collected by the KISTLER Dyno Ware software. The cooling of the work piece/ fixture/ machine tool/ cutting tool (WFMT) system was ensured by minimum quantity lubrication (MQL), where a TRIM ML26 lubricant was applied. The machining environment can be seen in Fig. 3.

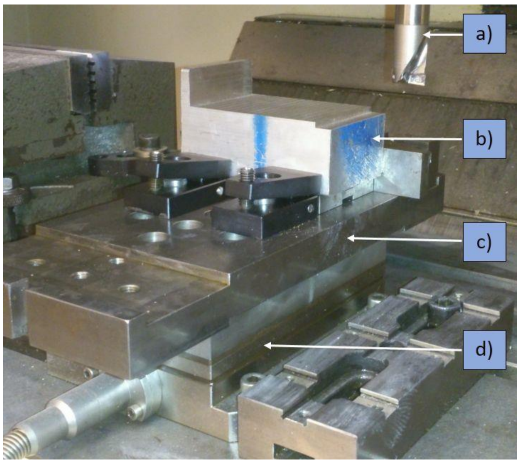

Figure 3. The machining environment a) cutting tool b) work piece c) fixture d) KISTLER load cell

Two experiment design methods, the full factorial (FF) and the central composite inscribed (CCI) methods have been examined. The experiments were designed via Minitab 17 mathematical-statistics software.

The cutting speed (v_c) and the feed rate (v_f) were chosen as factors. Based on the literature research [9-13], these are the factors which have the biggest impact on the machinability of aluminum alloy. When selecting the interpretation range of the factors, technological considerations (machine tool, cutting tool geometry, fixture and cutting tool stability, tool life) and economic limitations were also taken into account. The experimental design factor levels and the interpretation range of the variable factors are presented in Table 1.

TABLE I.
Factor levels of the variable factors

FF	v_c [m/min]	100	122	144	166	188	210	232	254	276	298	320
	v_f [mm/rot]	0.10	0.15	0.20	0.25	0.30	0.35	0.40				
CCI	v_c [m/min]	100	132	210	288	320						
	v_f [mm/rot]	0.10	0.14	0.25	0.35	0.40						

During the experiments the presence of other factors and noises also had to be taken into account. Values of these parameters have been fixed at one level to gain that their effect on the response variables do not change. The axial (a_p= 3mm) and the radial (a_e= 5mm) depth of cut (*DOC*) were fixed during the experiments, and only climb milling was applied.

III. Results and Discussion

The evaluation of the results was performed in Minitab 17 and Microsoft Excel software packages. On the obtained high frequency signal Gaussian frequency filtering was performed, where the actual point was calculated by its own surrounding's weighted average. A t_s=1 second section of the filtered signal was evaluated, so the cutting force components ($F_{x,max}$; $F_{y,max}$) and the resultant cutting force (F_{max}) was also calculated using the data of that particular section. In the following parts of the study these values (response variables) were used to describe the machinability of the aluminum alloy, draw the graphs and to analyze the statistical results as well. The statistical analysis has been performed at a 95% significance level.

The results and discussion part of the study is organized as follows: firstly, the results of full factorial (FF) experiment are shown and explained. Secondly the central composite inscribed (CCI) experiment is analyzed through the response surface methodology (RSM) and analysis of variance (ANOVA) techniques. Finally, the results of FF and CCI designs are compared and discussed based on the values of the response variables ($F_{x,max}$; $F_{y,max}$; F_{max}).

A. Results of Full Factorial (FF) experiments

77 experiments were carried out based on the FF design (cutting force with 11 levels and feed rate with 7 levels). The FF results are listed in Table 2., the response graphs can be seen in Fig. 4.

TABLE II.
EXPERIMENTAL VALUES OF FULL FACTORIAL DESIGN

No. [-]	v_c [m/min]	v_f [mm/rot]	FF $F_{x,max}$ [N]	FF $F_{y,max}$ [N]	FF F_{max} [N]
1	320	0.35	271.71	343.31	422.44
2	166	0.25	183.81	326.54	357.87
3	276	0.15	155.75	230.75	258.55
4	298	0.20	135.73	257.98	273.99
5	298	0.40	241.37	380.62	429.33
6	188	0.30	209.30	337.68	381.89
7	298	0.30	202.38	319.21	350.03
8	122	0.35	258.85	433.92	484.82
9	276	0.10	86.24	170.76	177.81
10	210	0.15	132.21	216.93	237.98
11	144	0.15	130.55	231.25	261.04
12	232	0.20	166.90	263.75	298.17
13	276	0.25	227.46	282.77	339.13
14	144	0.20	172.23	272.49	314.49
15	320	0.15	129.69	217.23	240.48
16	210	0.35	243.24	355.61	408.24
17	232	0.15	122.50	231.47	251.33
18	188	0.25	176.95	301.46	338.76
19	232	0.30	224.83	334.92	390.32
20	320	0.40	316.44	369.54	469.53
21	232	0.35	277.35	378.32	442.85
22	320	0.20	183.74	253.53	301.73
23	320	0.25	221.25	289.83	350.51
24	210	0.30	215.50	332.15	373.40
25	256	0.20	150.49	267.27	284.32
26	166	0.30	212.77	367.42	409.55
27	256	0.25	179.10	307.59	331.59
28	320	0.30	199.80	306.97	350.38
29	100	0.30	231.10	388.09	433.46
30	100	0.25	202.89	350.38	385.76
31	166	0.15	111.09	239.51	253.18
32	298	0.35	180.14	344.65	378.45
33	256	0.35	216.57	387.65	434.29
34	256	0.15	104.11	235.32	247.38
35	232	0.10	81.98	181.81	192.70
36	188	0.35	216.15	359.45	406.88
37	166	0.20	139.28	284.92	308.31
38	100	0.15	140.34	239.90	262.92
39	166	0.40	259.22	431.42	482.18
40	122	0.15	120.31	223.82	243.11
41	256	0.40	244.95	405.46	454.71
42	256	0.10	76.85	184.06	189.75
43	144	0.40	278.10	432.97	502.70
44	276	0.35	296.52	376.50	445.73
45	122	0.40	280.50	467.46	525.32
46	188	0.15	108.84	221.59	235.16
47	232	0.25	169.52	305.46	336.34
48	276	0.30	238.83	315.92	366.84
49	100	0.20	163.38	285.45	315.52
50	210	0.25	185.31	286.50	321.96
51	122	0.30	212.35	383.78	419.13
52	122	0.10	81.65	180.94	193.61
53	256	0.30	193.27	345.04	385.84
54	188	0.10	73.25	174.87	186.37
55	210	0.20	154.64	253.86	280.30
56	100	0.40	289.86	485.17	537.33
57	122	0.25	190.87	327.15	357.68
58	320	0.10	77.60	173.84	184.91
59	100	0.10	97.62	182.09	197.95
60	166	0.35	232.25	393.32	434.26
61	276	0.20	182.98	265.08	302.88
62	210	0.40	268.37	397.41	433.91
63	144	0.25	191.27	316.02	360.20
64	144	0.35	251.91	388.05	445.99
65	188	0.40	246.18	407.07	448.31
66	144	0.30	185.73	358.74	391.40
67	298	0.10	66.44	168.90	171.61
68	166	0.10	75.81	190.32	197.57
69	144	0.10	83.92	172.66	190.06
70	188	0.20	131.95	254.50	277.50
71	298	0.25	125.31	275.00	293.63
72	232	0.40	270.53	393.54	450.48
73	122	0.20	146.66	269.15	294.89
74	210	0.10	87.07	174.65	185.02
75	298	0.15	80.60	217.91	222.32
76	100	0.35	257.87	422.65	464.43
77	276	0.40	313.55	376.14	458.97

The response surfaces presented in Fig. 4. are constructed of planes with different steepness (conventional graphs without regression). The boundaries for these planes are provided by the set factor levels. On the graph the optimization parameters ($F_{x,max}$; $F_{y,max}$, F_{max}) are shown as functions of the variable factors (v_c, v_f).

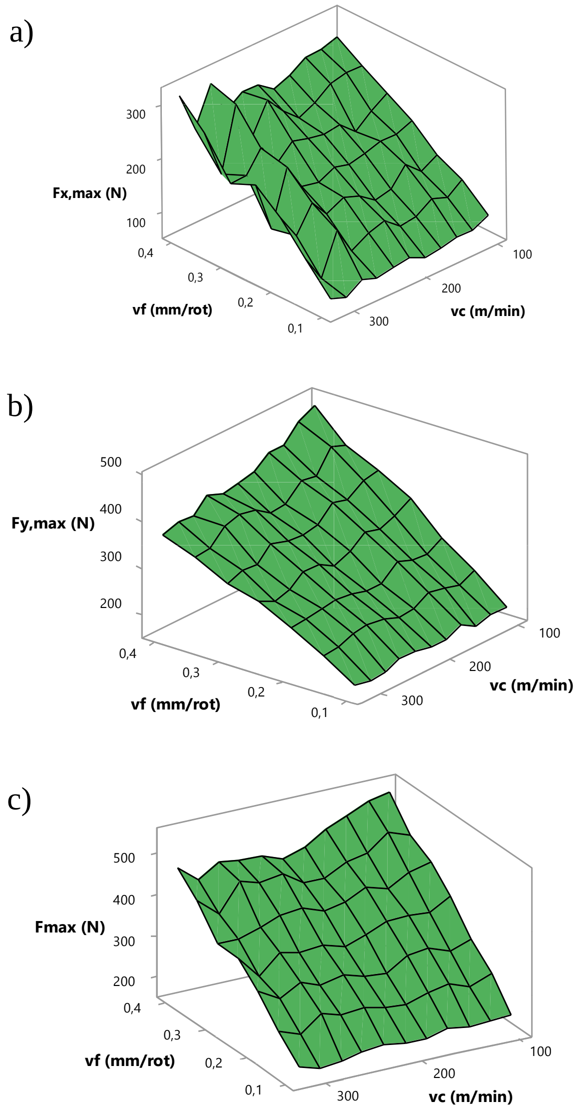

Figure 4. Effect of cutting speed (v_c) and feed rate (v_f) on the cutting force components used Full Factorial (FF) design a) $F_{x,max}$ b) $F_{y,max}$ c) F_{max}

As Fig. 4. shows, increasing the value of the feed rate increases all three response variables. However, the effect of the cutting speed is less significant. In the observed factor space, the highest response variables have been obtained at a high feed rate (0.4 mm/rot) and a low cutting speed (100 m/min), and the lowest ones at low feed rates (0.1 mm/rot). The main-impact and interaction plots are shown in Fig. 5.

Based on the ANOVA and the main effect plots of the F_{max} (Fig. 5) it is foreseeable that the feed rate has the most significant impact on the response variable. However, the effect of the cutting speed is low. Based on the interaction plot (Fig. 5) it can be observed that the lines on the graph are nearly parallel, so the cutting speed and the feed rate has a low interaction in the analyzed factor space.

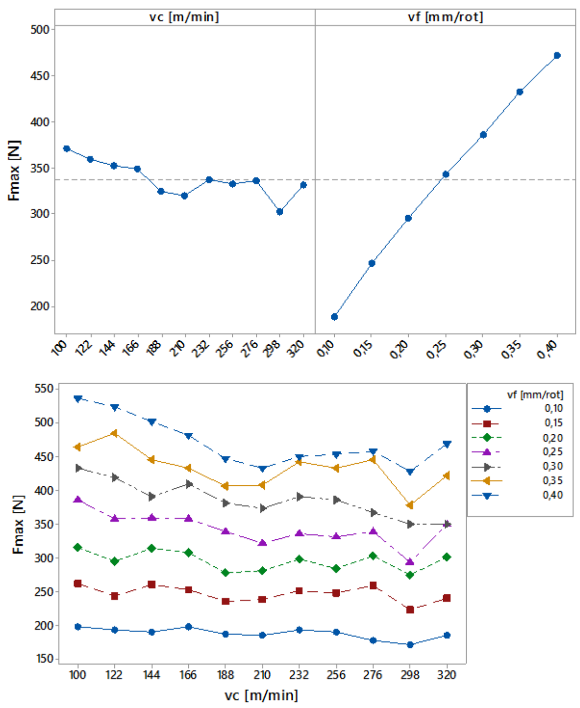

Figure 5. Main-impact and interaction plots used Full Factorial design a) main-impact on the F_{max} b) interaction on the F_{max}

B. *Results of Central Composite Inscribed (CCI) experiments*

13 experiments were carried out based on the CCI design (both of the factors with 5 levels). The CCI results are listed in Table 3., the response surfaces obtained by the RSM method are shown in Fig. 6.

TABLE III.
EXPERIMENTAL VALUES OF CENTRAL COMPOSITE INSCRIBED DESIGN

No. [-]	v_c [m/min]	v_f [mm/rot.]	CCI $F_{x,max}$ [N]	CCI $F_{y,max}$ [N]	CCI F_{max} [N]
1	210	0.100	95.95	178.91	191.44
2	288	0.356	238.59	330.45	376.62
3	210	0.250	193.23	289.20	334.22
4	100	0.250	206.32	354.61	394.58
5	132	0.356	255.29	445.30	506.84
6	210	0.250	197.35	292.36	338.70
7	210	0.250	197.72	293.77	339.32
8	210	0.250	196.51	300.03	344.98
9	210	0.250	198.36	298.95	344.79
10	288	0.144	125.35	203.34	212.78
11	132	0.144	108.78	237.98	258.00
12	210	0.400	293.47	396.92	466.32
13	320	0.250	208.92	289.50	336.71

The RSM-based mathematical models developed to predict the response variables are expressed by Eq. (2), (3) and (4).

The effects of the cutting speed and the speed rate observed through the CCI design are perfectly similar to the effects observed through the FF design, as visible in the response surfaces, in the Fig. 4 and Fig. 6.

$$F_{x,\max}(v_c, v_f) = -43.3 + 0.211 v_c + 1102 v_f + \\ + 0.000111 v_c^2 - 511 v_f^2 - 1.0006 v_c v_f \qquad (2)$$

$$F_{y,\max}(v_c, v_f) = 136.8 - 0.711 v_c + 1430 v_f + \\ + 0.002214 v_c^2 - 327 v_f^2 - 2.425 v_c v_f \qquad (3)$$

$$F_{\max}(v_c, v_f) = 87.8 - 0.494 v_c + 1838 v_f + \\ + 0.001722 v_c^2 - 707 v_f^2 - 2.57 v_c v_f \qquad (4)$$

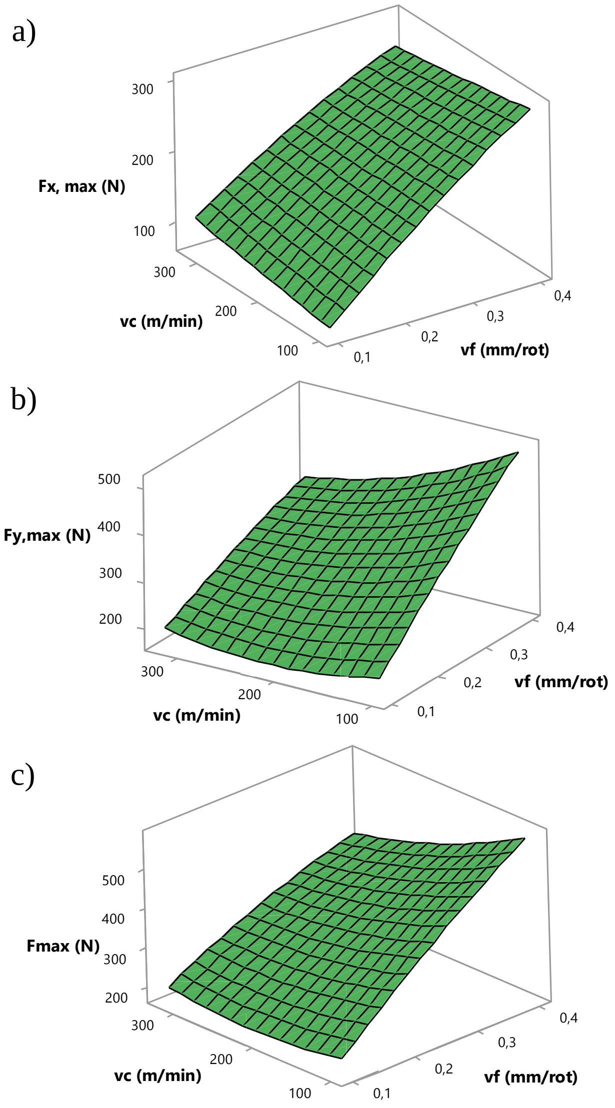

Figure 6. Effect of cutting speed (v_c) and feed rate (v_f) on the cutting force components used Central Composite Inscribed (CCI) design a) $F_{x,max}$ b) $F_{y,max}$ c) F_{max}

As illustrated in Fig. 6., increasing the value of the feed rate rises the cutting force components. However, the effect of the cutting speed is less significant. In the observed factor space, the highest and lowest response variables were obtained at the same factor levels like the levels with the FF design.

The ANOVA tables are listed in Table 4-6, the main-impact and interaction plots are shown in the Fig. 7. Based on the ANOVA table (Table 6) and the main effect plots of the *Fz* (Fig. 7) it is foreseeable that the feed rate has the most significant impact (F-Value: 317.69) on the response variable, followed by the cutting speed (F-Value: 0.01).

TABLE IV.
ANALYSIS OF VARIANCE TABLE FOR $F_{X,MAX}$

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	36849.8	7370.0	64.45	0.000
Linear	2	36327.9	18164.0	158.85	0.000
v_c	1	1.6	1.6	0.01	0.910
v_f	1	36326.3	36326.3	317.69	0.000
Square	2	245.2	122.6	1.07	0.393
$v_c v_c$	1	3.4	3.4	0.03	0.867
$v_f v_f$	1	230.2	230.2	2.01	0.199
Interaction	1	276.7	276.7	2.42	0.164
$v_c v_f$	1	276.7	276.7	2.42	0.164
Error	7	800.4	114.3		
Total	12	37650.2			

TABLE V.
ANALYSIS OF VARIANCE TABLE FOR $F_{Y,MAX}$

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	62003.5	12400.7	149.35	0.000
Linear	2	58936.0	29468.0	354.89	0.000
v_c	1	7294.8	7294.8	87.85	0.000
v_f	1	51641.2	51641.2	621.94	0.000
Square	2	1459.0	729.5	8.79	0.012
$v_c v_c$	1	1252.3	1252.3	15.08	0.006
$v_f v_f$	1	92.9	92.9	1.12	0.325
Interaction	1	1608.5	1608.5	19.37	0.003
$v_c v_f$	1	1608.5	1608.5	19.37	0.003
Error	7	581.2	83.0		
Total	12	62584.7			

Based on the interaction plot (Fig. 7) and the ANOVA tables it can be observed that the interaction terms between the cutting speed and feed rate are not considerable. Furthermore, the square terms of the RSM based models are also less significant (F-Values: 1.07 for $F_{x,max}$, 8.79 for $F_{y,max}$ and 3.43 for F_{max}).

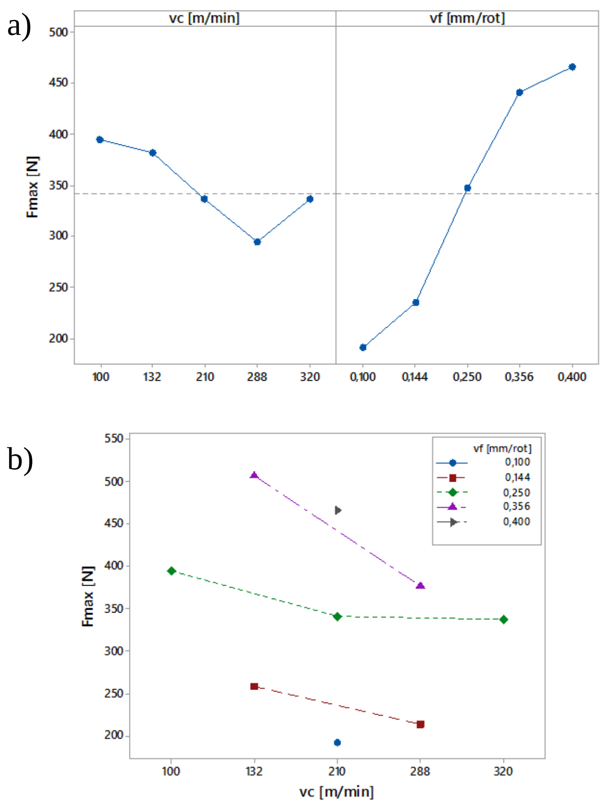

Figure 7. Main-impact and interaction plots used Central Composite Inscribed design a) main-impact on the F_{max} b) interaction on the F_{max}

TABLE VI.
ANALYSIS OF VARIANCE TABLE FOR F_{MAX}

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	91736.3	18347.3	91.69	0.000
Linear	2	88558.4	44279.2	221.29	0.000
v_c	1	8274.8	8274.8	41.35	0.000
v_f	1	80283.6	80283.6	401.23	0.000
Square	2	1371.2	685.6	3.43	0.092
$v_c v_c$	1	759.1	759.1	3.79	0.092
$v_f v_f$	1	438.3	438.3	2.19	0.182
Interaction	1	1806.6	1806.6	9.03	0.020
$v_c v_f$	1	1806.6	1806.6	9.03	0.020
Error	7	1400.7	200.1		
Total	12	93137.0			

C. Comparison of FF and CCI designs

The obtained response variables ($F_{x,max}$, $F_{y,max}$, F_{max}) from the two different experiment design methods (FF, CCI) were compared and analyzed in order to show the differences of the collected information and experiment time needs. The maximum and the minimum locations and values of the response variables were also determined and summarized in Table 7.

TABLE VII.
MAX. AND MIN. LOCATIONS, VALUES, DIFFERENCE BETWEEN THE FF AND CCI DESIGNS

	Full Factorial (FF)			Central Composite Inscribed (CCI)			Error		
	$F_{x,max}$ [N]	$F_{y,max}$ [N]	F_{max} [N]	$F_{x,max}$ [N]	$F_{y,max}$ [N]	F_{max} [N]	$F_{x,max}$ [%]	$F_{y,max}$ [%]	F_{max} [%]
Max value	316.44	485.17	537.33	297.71	510.52	574.90	6	-5	-7
Max loc. of v_c	320	100	100	100	100	100	69	0	0
Max loc. of v_f	0.4	0.4	0.4	0.4	0.4	0.4	0	0	0
Min value	66.44	168.90	171.61	73.94	173.93	182.76	-11	-3	-6
Min loc. of v_c	298	298	298	100	210	210	66	30	30
Min loc. of v_f	0.1	0.1	0.1	0.1	0.1	0.1	0	0	0

The comparative analysis of FF and CCI designs shows that the difference between the estimated locations of the max. and min. cutting speed is considerable (30% - 69%), as shown Table 7. This related to the main and interaction studies, since the factor has no significant effect on the responses, so its determination involves greater uncertainty.

However, the maximum and minimum locations of the feed rate is identical using both DOE methods. The error of the estimated max. and min. values of the response variables are less than 11% using both design of experiments.

There is a significant difference between the two design of experiment methods when considering the experiment time spent, because of the different amount of experiment setups. The net cutting time (when the cutting edges of the cutting tool are machining) was 2 minutes used the CCI method and 11 minutes when used the FF design. Consequently, the difference between the time spent of the DOE methods is 550%, in case of two factors. This difference can be even higher when there are more than two factors used in the experiment design.

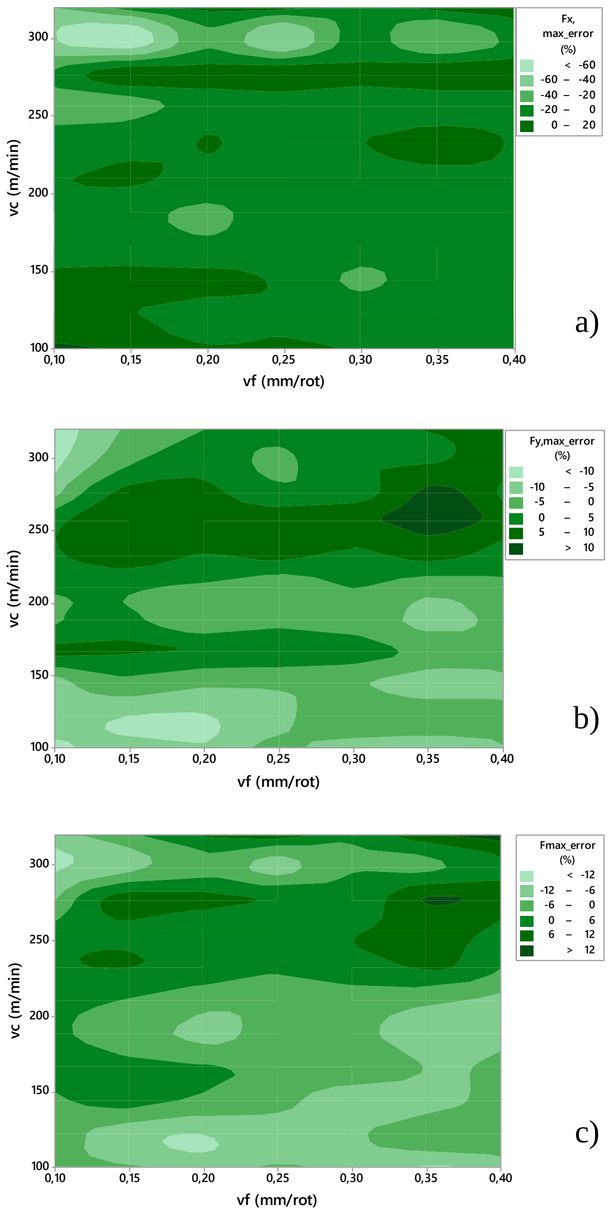

Figure 8. Error of response variables of Central Composite Inscribed design compared with the Full Factorial design ones a) $F_{x,max}$ b) $F_{y,max}$ c) F_{max}

The error of the results of the CCI design ((CCI-FF)/CCI) all over the factor space, is illustrated in Fig. 8. The error of CCI when estimating $F_{x,max}$ can be higher than 60%, as highlighted in Fig. 8 (a). However, the error of CCI when estimating $F_{y,max}$ and F_{max} response variables is less than 20%. The cutting speed increases the error of estimated response variables, but the effect of feed rate is not unambiguous.

IV. SUMMARY

Full Factorial (FF) and Central Composite Inscribed (CCI) designs of experiments were compared through machinability analysis of 6028 aluminum alloy in order to analyze their differences. The cutting speed and the feed rate were selected as factors, and their effect was investigated on the component of the cutting force (as response variables). The aim of our research was to carry out experiments using different experiment design methods, and then compare their information content. The following conclusions could be drawn:

- The greatest cutting force has been obtained by the lowest cutting speed and highest feed rate values, using both DOE methods.
- The effect of the cutting speed on the response variables is not significant, while the feed rate has a significant effect; however, there is no interaction between the variable factors. These statements are true for both experiment designs.
- For both experiment designs it can be concluded that the measurement uncertainty increases as the cutting speed increases, while it remains nearly constant with increasing feed rates.
- The max. and min. results of the CCI experiment design show a maximum 11% difference and a 18% less time spent when compared to the FF results. So the CCI method is more cost-effective, since it requires less experimental settings, and less time.

The aim of our future research is to compare other experiment design methods (Fig. 2) and analyze their time and cost effectiveness. This will help Machinability R&D Engineers in choosing optimal experimental designs.

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