Contradiction Resolution in the Adaptive Control of Underactuated Mechanical Systems Evading the Framework of Optimal Controllers

József K. Tar[†], János F. Bitó[†], Imre J. Rudas[‡]

[†]Antal Bejczy Center for Intelligent Robotics, [‡]Research and Innovation Center,
[†,‡]John von Neumann Faculty of Informatics, Institute of Applied Mathematics,
Óbuda University, H-1034 Budapest, Bécsi út 96/b, Hungary
E-mail: {tar.jozsef@nik., bito@, rudas@}uni-obuda.hu

Abstract: In the practice, precise and efficient control is needed for certain state variables of multiple variable physical systems in which the number of the independent control variables is less than that of the independent state variables. In such cases, either the propagation of certain state variables is completely abandoned or the concept of the Model Predictive Control (MPC) *is applied in which the model of the controlled system is embedded into the mathematical framework of the* Optimal Controllers*. This approach uses a* cost function *that summarizes the contributions of the frequently contradictory requirements. By minimizing this cost a kind of "compromise" is achieved. Whenever approximate and/or incomplete system models are available, the use of this controller is justified only for short time-intervals. The only way to reduce the accumulation of the effects of the modeling errors is the frequent re-design of the time horizon from the actual state as initial state that is done by the* Receding Horizon Controllers*. The more sophisticated* Adaptive Controllers *are designed by the use of Lyapunov's "Direct Method" that has a complicated mathematical framework that cannot easily be combined with that of the optimal controllers. As a potential competitor of the Lyapunov function-based adaptive controllers a* Fixed Point Transformation*-based approach was invented that in the first step transforms the the problem of computing the control signal into the task of finding an appropriate fixed point of a contractive map. The fixed point can be found by iteration in which the iterative sequence is generated by this contracting map. This method can be used for contradiction resolution without the minimization of any cost function by tracking the observable state components with time-sharing on a rotary basis.*

In the present paper a novel fixed point transformation is introduced. It is shown that this construction for monotonic response function of bounded derivative can guarantee global stability. Furthermore, the time-sharing-based method is demonstrated by the control of an underactuated 3 DoF Classical Mechanical system via numerical simulations.

Keywords: adaptive control; underactuated mechanical systems; fixed point transformations; optimal control; contradiction resolution; time-sharing;

1 Introduction

In practice, precise and efficient control is needed for certain state variables of multiple variable physical systems in which the number of the independent control variables is less than that of the independent state variables.

Typical examples are the *underactuated Classical Mechanical systems*, such as the *Translational Oscillations with an Eccentric Rotational Proof Mass Actuator (TORA)* that is a simplified model of a dual-spin spacecraft with mass imbalance. It serves as a "benchmark paradigm" for controller designers (e.g. [1]) for the control of which various controllers can be developed as a cascade and a passivity based controller in [2], a model-based controller using the *Tensor Product Form (TP)* in [3]. In [4] nine papers were published on the control of the TORA system in a special issue.

From the subject area of physiology, the illness called *Type 1 Diabetes Mellitus (T1DM)*, has various, more or less complex models. *Bergman's Minimal Model* as presented by Chee et Fernando in [5] has only three state variables. More complex models of the same phenomenon take into consideration more variables (e.g. [6]) that can be combined with digestion models as e.g. that in [7] work with 10 state variables. However, the only measurable variable is the blood glucose concentration while we have only one control signal, namely, the insulin ingress rate.

Another interesting area from the realm of nonlinear phenomena is the operation of the neurons. From the beginning of the 20th century various efforts were made to expound the spiking property of the neurons. From Lapicque's *"Integrate and Fire Neuron Model"* in 1907 to the quite sophisticated Hodgkin-Huxley model in 1952 [8] distinguishing between sodium, potassium and leakage channels. Various simplifications were also used. As examples the *Chua-Matsumoto Circuit* in 1984 [9] or the FitzHugh-Nagumo model in 1961 [10] can be mentioned. Each of these models is a multivariable system having nonlinear dynamic coupling between its variables to which only one control signal is available.

In a wider sense the above examples well represent the "underactuated systems". These systems have the important feature that makes it physically impossible to drive them through an arbitrary "trajectory" along which each state variable's precise position is prescribed in time. Controlling only one state variable and letting the other ones propagate as they want generally cannot be an acceptable option. It is more expedient to somehow "distribute" the tracking error over the various state variables that evidently may be a contradictory task. A plausible solution for contradiction resolution is the minimization of a cost functional that is constructed as a sum of the errors to be kept at bay as well as some other terms that express some limitations of the controllers or other extra requirements. In general this problem appears as the Hamilton-Jacobi-Bellman equation that can be solved by *Dynamic Programming* that consumes up a lot of computational power [11]. It uses the principle of optimality of subproblems and applies tabulation in the state space to compute recursively a feedback control. The *Indirect Methods* are related to the variational principles of Classical Mechanics with the introduction of the *Lagrange Multipliers* as "co-state" variables. The direct methods transform the infinite optimal control problem into a finite dimensional *Nonlinear Programming Problem (NLP)*. Simply treatable problem is obtained only if the goal functional has very special form as e.g. in the case of the *Linear Quadratic Regulator* [12, 13] in which, the controlled system is *Linear Time Invariant (LTI)* and the cost function has quadratic structure. This makes the problem tractable by using Riccati differential or algebraic equations, depending on the role of the terminal conditions. The simplicity means that a separate solution becomes available for the co-state and the state variables. In more general cases the possibility for this separation ceases and *state-dependent Riccati equations* appear (e.g. [14]). These approaches normally are based on the globally linearizable form and apply state-dependent weighting matrices in the LQR form.

Whenever the available models are not precise enough, the time-horizon for the controller design cannot be too long. To evade the accumulation of the effects of the modeling errors the so calculated control signal can be applied only for a short time-horizon, the new initial conditions have to be measured and a redesign has to be initiated for the next short period (the *Receding Horizon Control* that appeared in the late seventies of the past century in relation with industrial applications [15]).

An alternative error-compensation possibility is the creation of an *Adaptive Controller*. The adaptive controllers traditionally are designed by the use of Lyapunov's "Direct Method" that he elaborated in his PhD dissertation in 1892 [16, 17] when he investigated the stability of motion. The main idea was that in spite of the fact that normally, the solution of coupled nonlinear differential equations cannot be expressed in closed analytical form, without knowing the details of the motion it became possible to determine its stability. The *Adaptive Inverse Dynamics Controller (AIDC)* and the *Adaptive Slotine-Li Controller (ASLC)* for robots in the nineties were developed by the use of this technique [18]. The method seems to be prevailing nowadays, in the design of the *Model Reference Adaptive Controllers (MRAC)*, too (e.g. [19, 20, 21]). Normally, this method can guarantee *global stability* without revealing any details of the controlled motion. In general whole set of adaptive control parameters can result in global stability with different "transients" of the controlled motion. Whenever these details are important the adaptive control parameters can be tuned to improve the transient behavior e.g. by evolutionary methods (e.g. [22, 23]). The application of Lyapunov function in adaptive controllers need very skilled control designers since for each particular control task one has to construct an individual Lyapunov function and has to prove the non-positivity of its time-derivative. One source of the mathematical difficulties may be that this method is based on *satisfactory conditions* and it may prescribe much more restrictions, than necessary. Another problem is that this approach does not seem to be easily integrable with the mathematical framework of the optimal controllers.

Another practical approach that can evade the complexity of the Lyapunov function based design is the use of *Robust Variable Structure/Sliding Mode Controllers (VS/SM)*. The very simple idea originates from the Soviet Union in the 1960s and became known to the western world later (e.g. [24, 25, 26]). Its main point was that in the possession of an approximate model only, and under the influence of unknown external disturbances, for a kinematically prescribed trajectory tracking, it is impossible to calculate the appropriate control signals. Instead of that the concept of the *error metrics* was so introduced that driving it to and keeping it in the vicinity of zero make the trajectory tracking error converge to zero. The great advantage of this solution is its simplicity and is easily realizable. Its drawback is that the control signal can sharply vary, when the components of the error metrics cross the value zero. The so generated *chattering* can excite "not modeled" subsystems unexpectedly. Normally this effect can be eliminated by "softening" the switching rule that may reduce the precision of the trajectory tracking.

With the aim of evading the difficulties of the Lyapunov function-based techniques and maintaining the simplicity of the VS/SM controllers without their aptitude for chattering, a novel adaptive controller-design methodology was suggested in 2009 [27, 28], that directly tries to realize a purely kinematically prescribed trajectory tracking property of the controlled motion by studying the response of the controlled system in the given control situation. This approach, at first, converts the control task into a *fixed point problem* that iteratively can be solved afterwards. The use of this idea goes back to the 17th century (the *"Newton-Raphson Algorithm"* [29]) and obtains great attention even in our days (e.g. [30, 31]). In 1922 Banach generalized this fixed point method to quite abstract linear, normed, complete metric spaces [32]. This iteration-based approach is essentially different to the method of *Iterative Learning Control (ILC)* that was elaborated for robots repetitively executing the same task (e.g. [33, 34, 35, 36, 37]). The original transformation introduced in [27] was called *Robust Fixed Point Transformation (RFPT)* that contained only three adaptive control parameters. While in several control applications, fix settings of these parameters was found to be satisfactory, for other cases, complementary tuning strategies were elaborated, for tuning only, its one adaptive parameters [38, 39, 40]. Attempts were also made to modify the fixed point transformation used for transforming the control task into a fixed point problem [41, 42, 43].

The method was found to be appropriate for various control tasks, via simulations, as e.g. chemical reactions [44, 45], the FitzHugh-Nagumo neuron model [46], the Hodgkin Huxley neuron [47], Chua-Matsumoto circuit [48], and various diabetes models [49, 50].

In [46] and [48] the idea of replacing the cost-functional-based optimal control approach for the resolution of the contradictions regarding the prescriptions for the tracking precision of the various state-variables of *underactuated systems* by time-sharing on a rotary basis already arose. The aim of the present paper is to show that this idea can work in the case of an underactuated Classical Mechanical system, the TORA model. In the simulations an improved version of the fixed point transformation suggested in [43] was used.

2 The Fixed Point Transformation-based Approach

This approach assumes that we have the *approximate model* of the dynamic system to be controlled. The control actions are calculated with its help based on some kinematically expressed trajectory error reduction by comparing the *desired* *response* r^{Des} and the *actually observed response* r^{Act} of the controlled system. Due to modeling imprecisions and unknown external disturbances normally $r^{Act} \neq r^{Des}$. For the given control situation a *"response function"* can be observed that sets a relationship between the desired and the observed responses as $r^{Act} = f\left(r^{Des}\right)$. The core of the fixed point transformation-based technique is the deformation of the input of the response function from r^{Des} to $r_\star$ using *Banach's Fixed Point Theorem* [32] in order to achieve the situation $r^{Act} \equiv f(r_\star) = r^{Des}$.

2.1 Antecedents

For the purpose of obtaining the appropriate deformation in [27] for SISO systems the transformation function was introduced as

$$r_{n+1} = (r_n + K_c)\left[1 + B_c \tanh\left(A_c\left\{f(r_n) - r^{Des}\right\}\right)\right] - K_c \tag{1}$$

with $K_c, B_c, A_c \in \mathbb{R}$ parameters. This construction had two fixed points for a monotonic response function $f(r)$: a trivial one at $r = -K_c$, and the solution of the control task $r = r_\star$. In [40] it was shown that for monotonic $f(r)$ and fixed K_c and B_c as $|A_c|$ was increased from zero the fixed point at $r = -K_c$ always behaved as a *monotonic repulsive* one, while the other one $r = r_\star$ at first was *monotonic attractive*, then it turned into *oscillatory attractive* before turning to an *oscillatory repulsive*. The session of the monotonic attractive behavior did not risk the stability of the controller, it was observed by *model-independent observers* and was used for tuning A_c in order to avoid the occurrence of the regime of bounded chaotic oscillations. On this reason these oscillations were called *Precursor Oscillations* in [40, 51]. The chaotic behavior was studied in [52, 53, 51] and it was shown that it was generated by the co-operation of two repulsive fixed points. It was found that in general it does not risk the precision of the trajectory tracking, and its great chattering can be reduced.

In [43] the *Sigmoid Generated Fixed Point Transformation* was suggested that was constructed of a monotonic increasing, bounded and smooth $g(x) : \mathbb{R} \mapsto \mathbb{R}$ *sigmoid* function. For some $K > 0$ and $D > 0$ positive parameters the iterative sequence $\{x_0, \ldots, g(x_n) - K = g(x_{n+1} - D), \ldots\}$ was generated that lead to the function $F(x) \stackrel{def}{=} g^{-1}(g(x) - K) + D$ where the inverse function of $g()$ is denoted by $g^{-1}()$. The *fixed point* of $F(x)$ is the solution of the equation $F(x_*) = x_*$. This function was used in [43] for the generation of the sequence of the *deformed inputs* as

$$r_{n+1} = G(r_n) \stackrel{def}{=} F\left(A_c\left[f(r_n) - r^{Des}\right] + x_*\right) + r_n - x_* \tag{2}$$

where $A_c \in \mathbb{R}$ stands for a parameter, and normally, in the control applications $r_0 \stackrel{def}{=} r_0^{Des}$, that is the *desired response* in the initial control cycle. Obviously, if $r_\star$ is the solution of the control task, i.e. $f(r_\star) - r^{Des} = 0$ then $G(r_\star) = r_\star$. Since $F(x_*) = x_*$, this solution is a fixed point of the function G. In order to guarantee the convergence of the series $\{r_n\}$ function G must be contractive, i.e. the relation

$\left|\frac{dG}{dr}\right| < 1$ must be valid. It was shown that this construction can have two fixed points as the originally suggested RFPT has, can produce the *precursor oscillations*, and can be used for adaptive control purposes.

However, this construction may have difficulties. Both constructions worked only with *bounded region of attraction* around $r_\star$ that formally cannot guarantee *global stability*. It is clear that the graph of the original bounded function $g(x)$ was *shifted down* in $g(x)-K$, and it was shifted to the right by $g(x-D)$. It was assumed that for the first element of the iteration x_0 there exists x_1 for which $g(x_0)-K = g(x_1-D)$. This may be possible for several x_0 values but not necessarily for each $x_0 \in \mathbb{R}$. For instance, if $g(x) \stackrel{def}{=} \tanh(x)$, $g(x)-K \in (-1-K, 1-K)$ and $g(x-D) \in (-1,1)$, therefore there exist x_0 to which no x_1 belongs. In the present paper this deficiency, is eliminated, by the introduction of a transformation generation technique, using *"Stretched Sigmoid Functions". It also will be shown that for SISO systems, of monotonic response functions, with bounded derivatives, this construction, can guarantee global stability.*

2.2 Stretched Sigmoids Generated Fixed Point Transformation (SSGFPT)

This fixed point transformation is generated by sigmoid functions $g(x)$, and $h(x)$ as follows:

$$g(x) \stackrel{def}{=} \tanh(x) - K_c \ , \tag{3a}$$

$$h(x) \stackrel{def}{=} \left(1 + \frac{K_c + J_c}{2}\right) \tanh\left(\frac{x - D_c}{1 + \frac{K_c + J_c}{2}}\right) - \frac{K_c + J_c}{2} \ , \tag{3b}$$

$$g(x_0) = h(x_1), \dots, g(x_n) = h(x_{n+1}), \dots \ , \tag{3c}$$

$$x_{n+1} = h^{-1}\left(g(x_n)\right) \equiv F(x_n) \ , \tag{3d}$$

in which $K_c, J_c,$ and $D_c > 0$, and $h^{-1}(x)$ denotes the inverse of the monotonic function $h(x)$. For the parameter settings $K_c = 0.5$, $J_c = 0.2$, and $D_c = 0.6$ this iteration is exemplified by Figs. 1, 2.

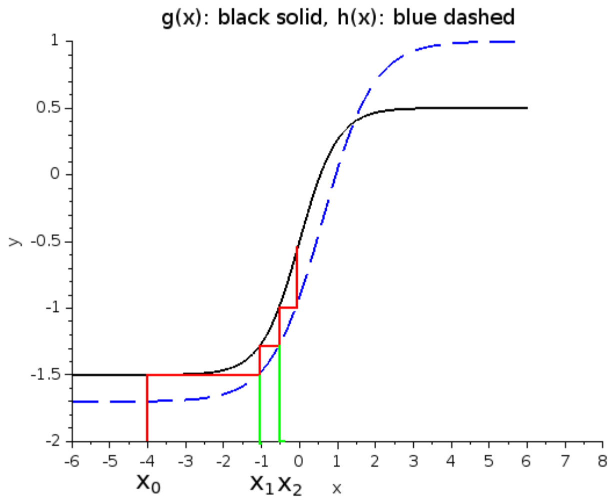

Figure 1
Generation of the fixed point transformation by the functions $g(x)$ and $h(x)$ in (3)

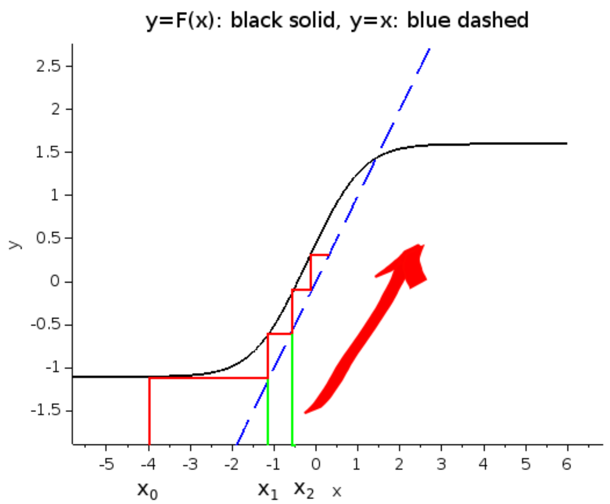

Figure 2
Generation of the fixed point transformation by the function $F(x)$ in (3)

Since $\forall x \in \mathbb{R}$ $g(x) \in (-1-K_c, 1-K_c)$, and $h(x) \in (-1-K_c-J_c, 1)$, the iteration defined in (3) always converges to $x_\star$.

The here suggested solution has considerable advantages in comparison with the RFPT transformation published in [27] or the new variants suggested in [42, 43]:

if the response function of the controlled system is monotonic and $\left|\frac{\mathrm{d}f}{\mathrm{d}r}\right|$ is bounded the function in (2) has only a single attractive fixed point over the whole real axis $\mathbb{R}$. Furthermore, due to the saturation of $g(x)$ and $h(x)$ it produces very fast convergence if the distance $|x - x_*|$ is great, and provides acceptable convergence speed in the vicinity of x_*. Due to the strongly saturated nature of $g(x)$ in (2) relatively great value for the parameter A_c can be used that also speeds up the convergence in the "beaked" structure near x_*. In the sequel the application of this fixed point transformation is exemplified in the case of an underactuated, extended TORA model.

3 Adaptive Optimal Control based on Time Sharing and SSGFPT

In this section the application of SSGFPT will be exemplified by the use of a TORA variant we already considered in [40, 43, 42]. At first the dynamic model is explained.

3.1 The Dynamic Model of the TORA System

The model consists of a cart, a pendulum (practically a beam) and a dial that can be rotated around an axle attached to the end of the beam. Its equations of motion under full actuation are are given in (4) with the dynamic parameters defined in Table 1. The generalized coordinates of the system are $q_1\,[rad]$ that describes the rotation of the beam with respect to the vertical direction, $q_2\,[rad]$ that is the rotation angle of the dial with respect to the beam, and $q_3\,[m]$ that corresponds to the translation of the cart in the horizontal direction. The generalized force components are $Q_1\,[N\cdot m]$, $Q_2\,[N\cdot m]$, and $Q_3\,[N]$. In the underactuated mode of operation it is assumed that $Q_2 \equiv 0$, and $Q_3 \equiv 0$, i.e. only a single control signal Q_1 can be used for controlling the motion of the axles q_1, q_2, and q_3.

$$\begin{bmatrix} (mL^2+\Theta) & \Theta & mL\cos(q_1) \\ \Theta & \Theta & 0 \\ mL\cos(q_1) & 0 & (m+M) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \\ + \begin{bmatrix} -mLg\sin(q_1) \\ 0 \\ -mL\sin(q_1)\dot{q}_1^{\,2} \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \tag{4}$$

[In the model the mass of the beam was neglected. Furthermore, it was assumed that the dial is connected to the beam by axle q_2 at its mass center point. These facts explain certain simplifications that are present in (4).]

In the sequel the idea of the adaptive optimal control is expounded.

Table 1
The parameters of the approximate model and that of the actually controlled system's model

Parameter	**Approximate**	**Exact**
$M\,[kg]$ mass of the cart	1.5×5	5
$L\,[m]$ length of the beam	1.5×1	1
$m\,[kg]$ mass of the dial	1.5×2	2
$\Theta\,[kg \cdot m^2]$ inertia momentum of the dial	1.5×6	6
$g\,[m \cdot s^{-2}]$ gravitational acceleration	1.5×9.81	9.81

3.2 Application of Time Sharing in Adaptive Optimal Control

In our system a single active control torque Q_1 can be used for controlling the motion of q_1, q_2, and q_3. It is evidently impossible to track some *arbitrarily prescribed nominal trajectory* $[q_1^N(t), q_2^N(t), q_3^N(t)]^T$. In the practice, the *tracking imprecisions have to be distributed over the three coordinates*. In the classical solutions as in the LQR controller (e.g. [13]) a cost function is constructed of the tracking errors and the cost functional that is obtained as its integral is minimized. Some limitation for the control forces can be built in the cost functional, too. The minimization can be executed by the use of the *Riccati equations* in the simpler cases, or by *nonlinear programming* in the more general ones.

In our approach the cost function is completely eliminated according to the idea of time-sharing. The operation time of the controller is divided into disjoint time intervals in which simultaneously the motion of only one coordinate is controlled while the other ones can propagate as they "want". In the next session the controller tries to keep at bay one of the other coordinates, and so on.

When the motion of q_1 is under control by the use of the last two equations of the matrix form in (4) $\ddot{q}_2$ and $\ddot{q}_3$ can be expressed by $\ddot{q}_1$, and these expressions can be substituted into the first equation with the application of the available approximate model parameters denoted by the subscript "a":

$$Q_1 = \left(m_a L_a^2 - \frac{m_a^2 L_a^2 \cos q_1^2}{m_a + M_a} \right) \ddot{q}_1 + \frac{(m_a L_a)^2 \cos q_1 \sin q_1 \dot{q}_1^2}{m_a + M_a} - m_a L_a g_a \sin q_1 \quad . \tag{5}$$

When the coordinate q_2 is under control from the third equation $\ddot{q}_3$ can be expressed by $\ddot{q}_1$, and from the second equation $\ddot{q}_1$ can be expressed by $\ddot{q}_2$. These values can be substituted into the first equation to obtain the appropriate control torque as

$$Q_1 = -\left(m_a L_a^2 - \frac{m_a^2 L_a^2 \cos q_1^2}{m_a + M_a} \right) \ddot{q}_2 + \frac{(m_a L_a)^2 \cos q_1 \sin q_1 \dot{q}_1^2}{m_a + M_a} - m_a L_a g_a \sin q_1 \quad . \tag{6}$$

Finally, when q_3 is under control, from the third equation $\ddot{q}_1$ can be expressed with $\ddot{q}_3$. With its use from the second equation $\ddot{q}_2$ can also be expressed by $\ddot{q}_3$. These quantities have to be substituted into the 1st equation to obtain Q_1 as:

$$Q_1 = \left(-\frac{(m_a+M_a)L_a}{\cos q_1} + m_a L_a \cos q_1\right)\ddot{q}_3 + \frac{m_a L_a^2 \sin q_1 \dot{q}_1^2}{\cos q_1} - m_a g_a L_a \sin q_1 \quad . \tag{7}$$

To each session some time-slot was allocated in the simulations detailed in the sequel.

4 Numerical Calculations

4.1 Setting the Parameters of the Numerical Simulations

The numerical simulations were made by the use of the SCILAB (ver. 5.5.2) and its graphically programmable package XCOS. The kinematically prescribed trajectory tracking contained a PD-type feedback defined by (8) using a *"time-constant of tracking"* $\Lambda > 0$:

$$e(t) \stackrel{def}{=} q^N(t) - q(t) \text{ tracking error} \quad , \tag{8a}$$

$$\left(\Lambda + \frac{\mathrm{d}}{\mathrm{d}t}\right)^2 e(t) = 0 \text{ desired behavior leading to} \tag{8b}$$

$$\ddot{q}^{Des} = \ddot{q}^N + 2\Lambda\dot{e} + \Lambda^2 e \quad . \tag{8c}$$

The numerical values are given in Table 2. Whenever the controlled axis was changed the adaptivity was switched off for a short period defined by the parameter *"time-slot of inactive adaptivity"*. This was necessary for "clearing the memory" of the adaptive controller that had inadequate antecedents since the past data at axle switching belonged to the previously controlled axle.

Table 2
The parameters of the adaptive controller and the numerical simulator

Parameter	**Value**
$\Lambda\,[s^{-1}]$	4
$K_c\,[nondimensional]$	0.5
$D_c\,[nondimensional]$	0.6
$J_c\,[nondimensional]$	0.2
$x_*\,[nondimensional]$ (dependent)	1.435116
$A_c \in \mathrm{I\!R}^3[\frac{s^2}{rad}, \frac{s^2}{rad}, \frac{s^2}{m}]^T$	$[-0.75, -2, -2]^T$
$\delta t\,[s]$ Cycle time	10^{-3}
$\Delta t\,[s]$ Time-slots for q_1, q_2, and q_3	$[2, 2, 1]^T$
$\tau_{NA}\,[s]$ Time-slot of inactive adaptivity	5×10^{-3}
Numerical integrator's method	Runge-Kutta 4(5) of SCILAB ver. 5.5.2
Maximum allowed time step in integration	Automatic

4.2 Simulation Results

The comparison of the operation of the non-adaptive and adaptive controllers can be done by using Figs. 3–5.

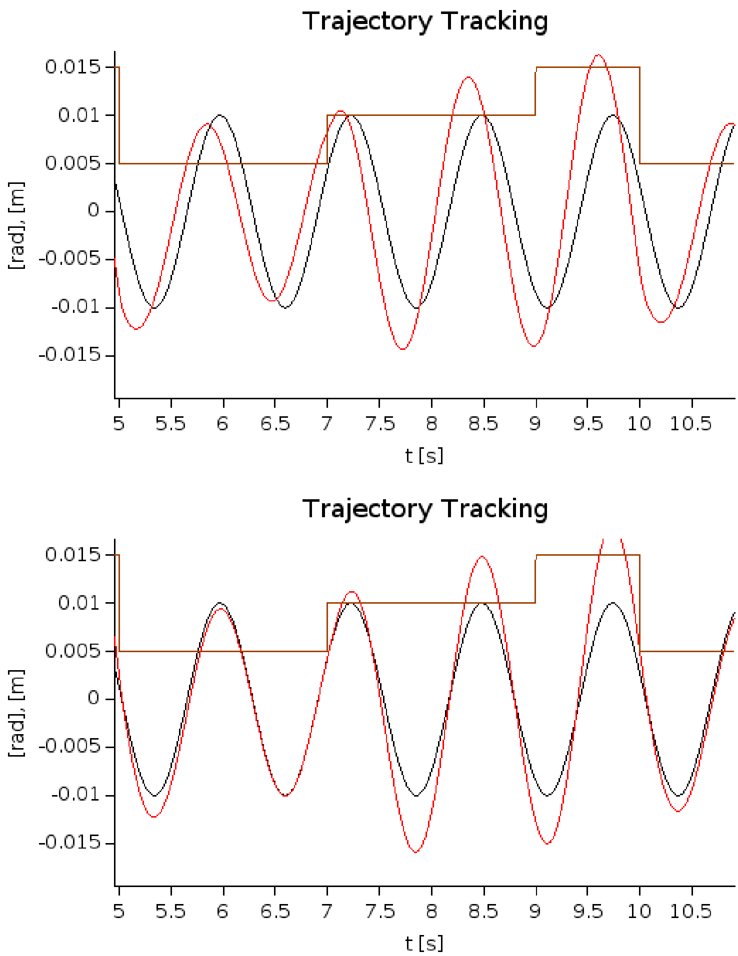

Figure 3
Tracking of q_1 in the non-adaptive (upper chart) and the adaptive (lower chart) cases: q_1^N: black, q_1: red lines, the time slots are indicated by the step function (brown line): increasing values belong to q_1, q_2, and q_3, respectively

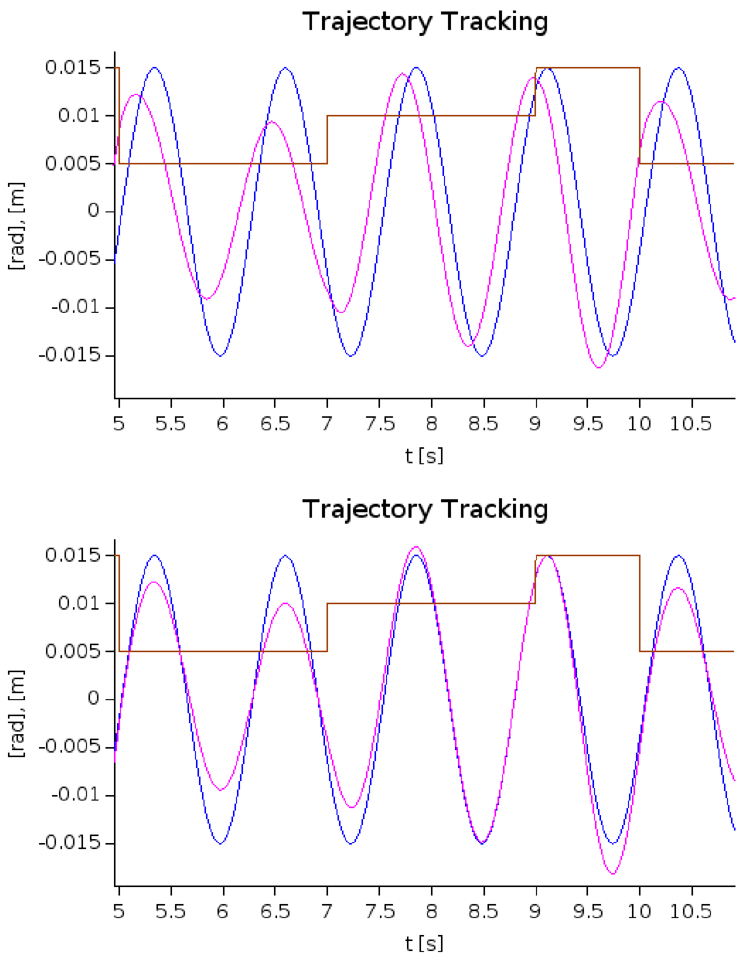

Figure 4
Tracking of q_2 in the non-adaptive (upper chart) and the adaptive (lower chart) cases: q_2^N: blue, q_2: magenta lines, the time slots are indicated by the step function (brown line): increasing values belong to q_1, q_2, and q_3, respectively

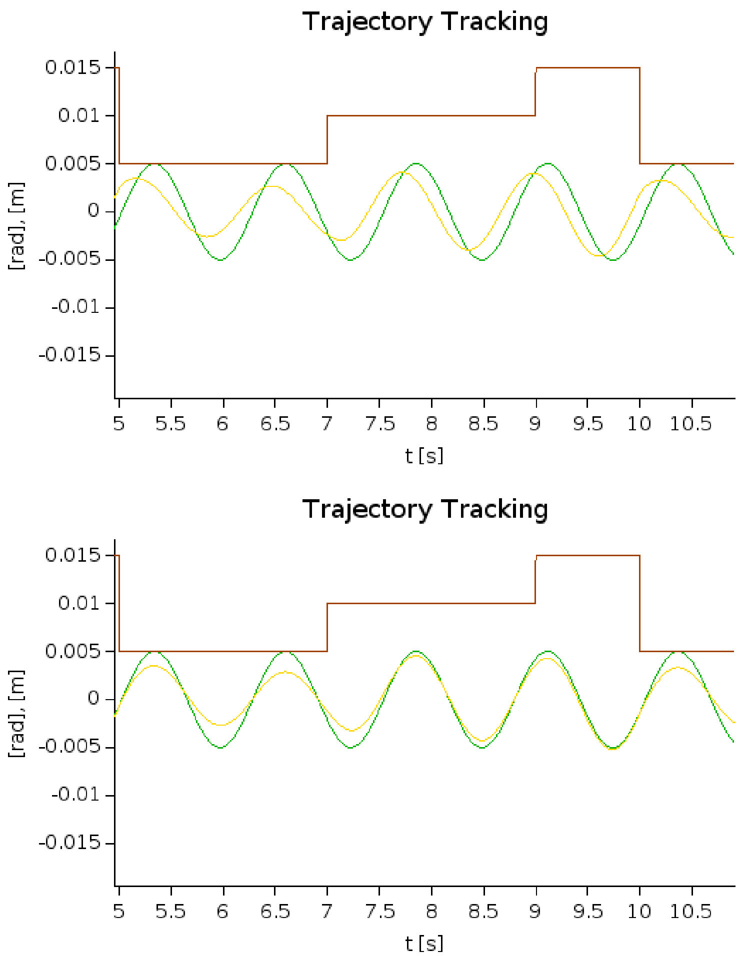

Figure 5
Tracking of q_3 in the non-adaptive (upper chart) and the adaptive (lower chart) cases: q_3^N: green, q_3: ocher lines, the time slots are indicated by the step function (brown line): increasing values belong to q_1, q_2, and q_3, respectively

More details are revealed by Figs. 6–8, displaying the tracking errors versus time. It is evident that within its own time-slot each of the adaptively controlled axle was adjusted to track the nominal motion that was prescribed to it. In the non-adaptive slots they left the nominal trajectory. However, due to the rotation of the adaptive slots the motion of each axle was kept in the vicinity of the nominal trajectory.

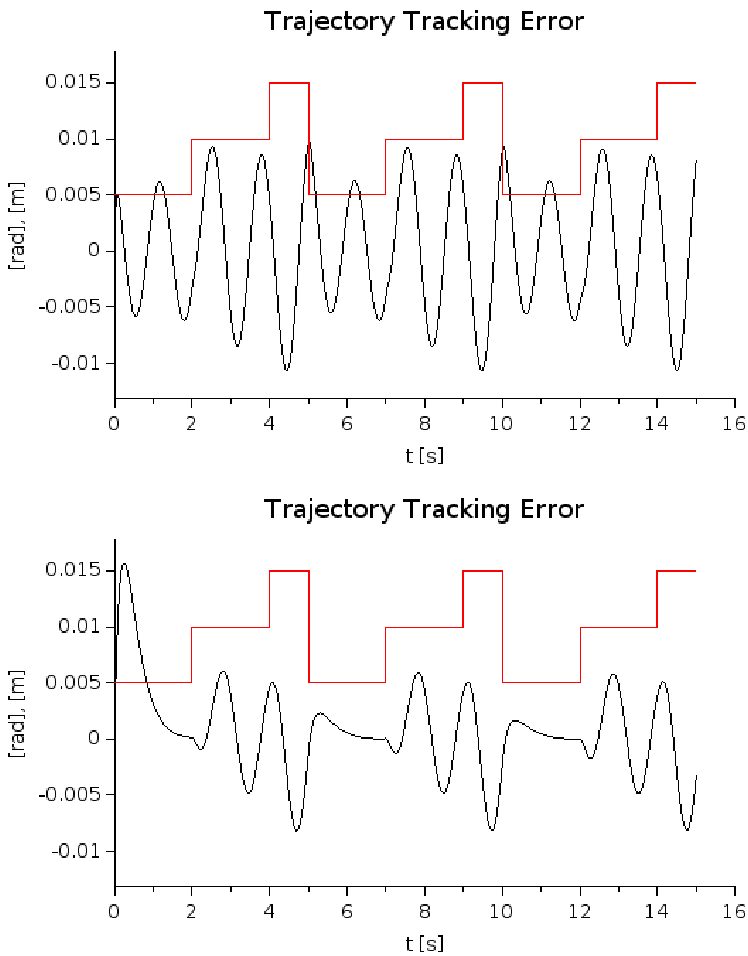

Figure 6
Tracking error of q_1 in the non-adaptive (upper chart) and the adaptive (lower chart) cases: $q_1^N - q_1$: black line, the time slots are indicated by the step function (red line): increasing values belong to q_1, q_2, and q_3, respectively

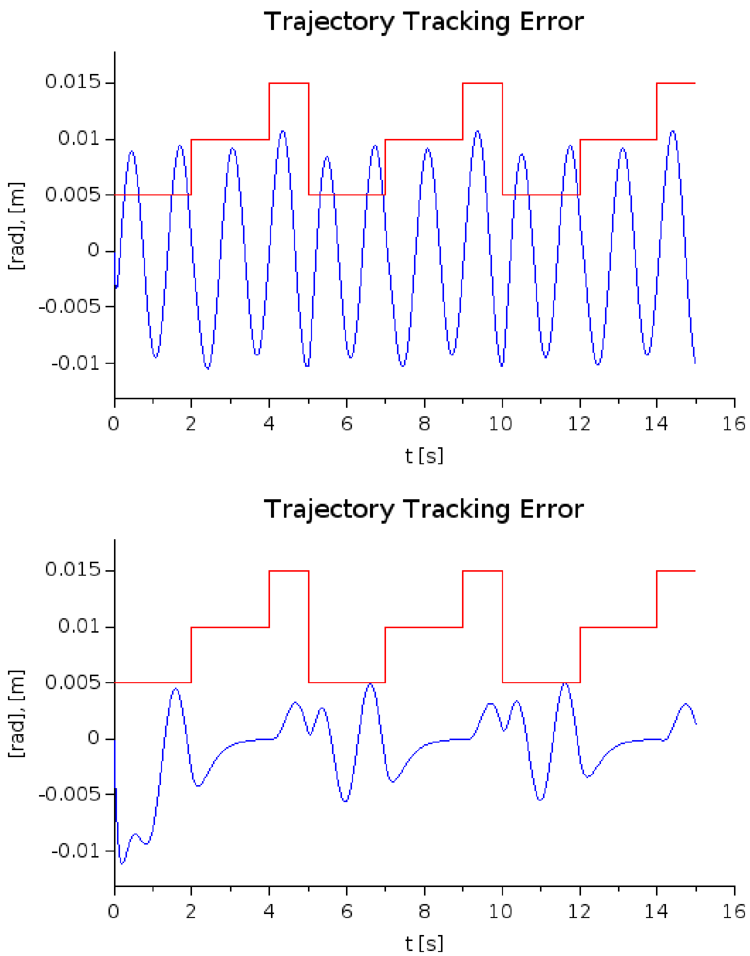

Figure 7
Tracking error of q_2 in the non-adaptive (upper chart) and the adaptive (lower chart) cases: $q_2^N - q_2$: blue line, the time slots are indicated by the step function (red line): increasing values belong to q_1, q_2, and q_3, respectively

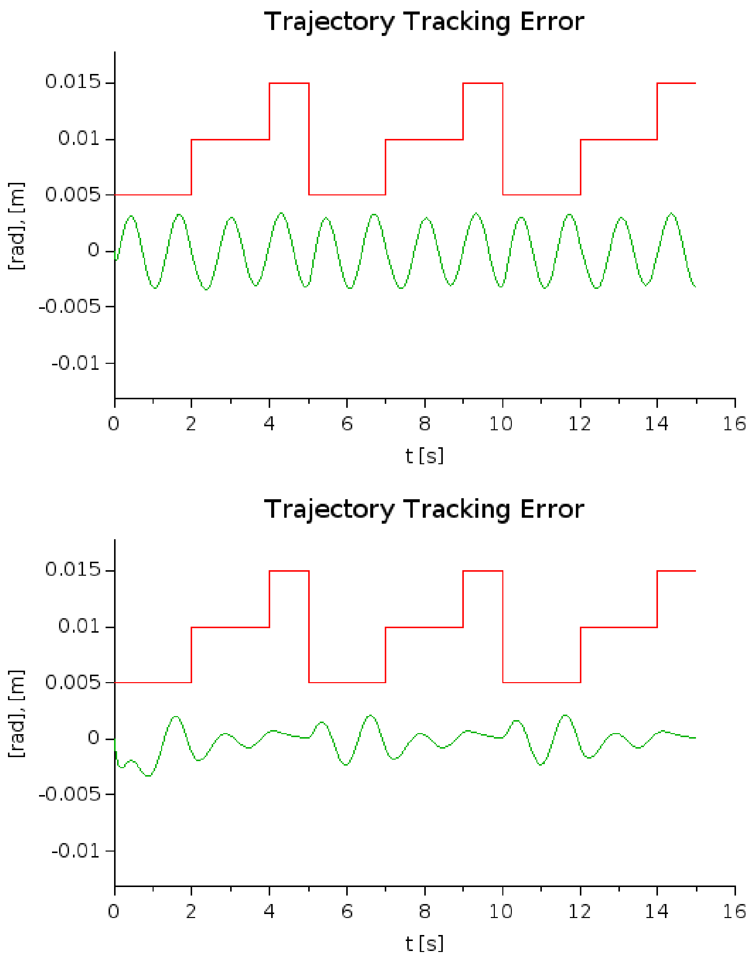

Figure 8
Tracking error of q_3 in the non-adaptive (upper chart) and the adaptive (lower chart) cases: $q_3^N - q_3$: green line, the time slots are indicated by the step function (red line): increasing values belong to q_1, q_2, and q_3, respectively

The generalized force Q_1 exerted by the controller is given in Fig. 9. The adaptive and the non-adaptive cases worked with control torques within the same order of magnitude.

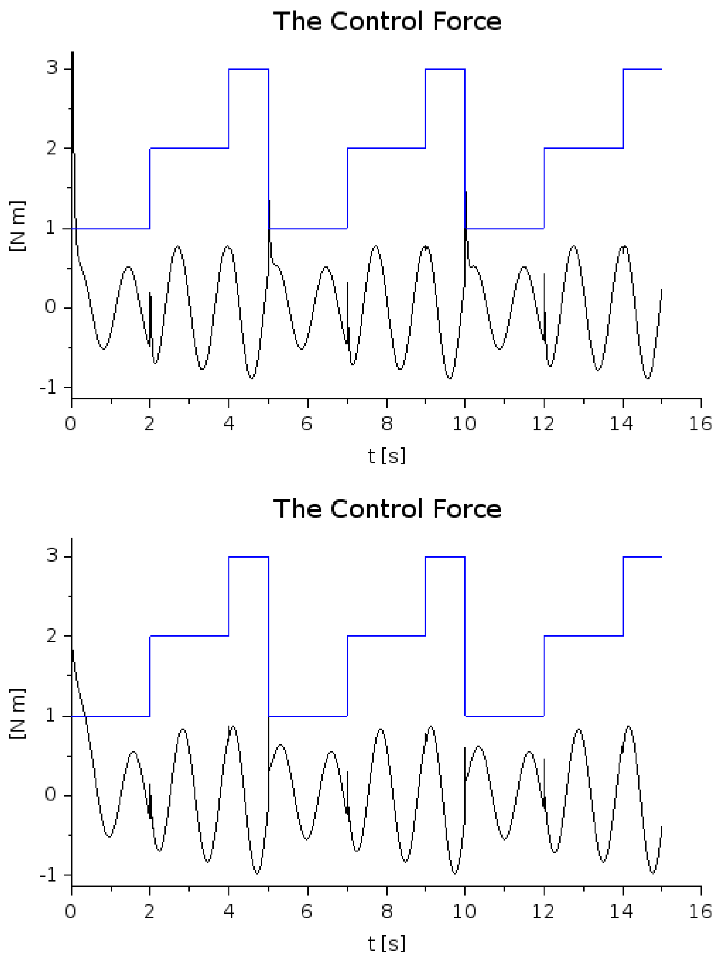

Figure 9
The control torque Q_1 in the non-adaptive (upper chart) and the adaptive (lower chart) cases; the time slots are indicated by the step function (blue line): increasing values belong to q_1, q_2, and q_3, respectively

The operation of the controller can be understood by considering the "desired" and the "realized" second time-derivatives of the generalized coordinates, as they are given in Figs. 10–12.

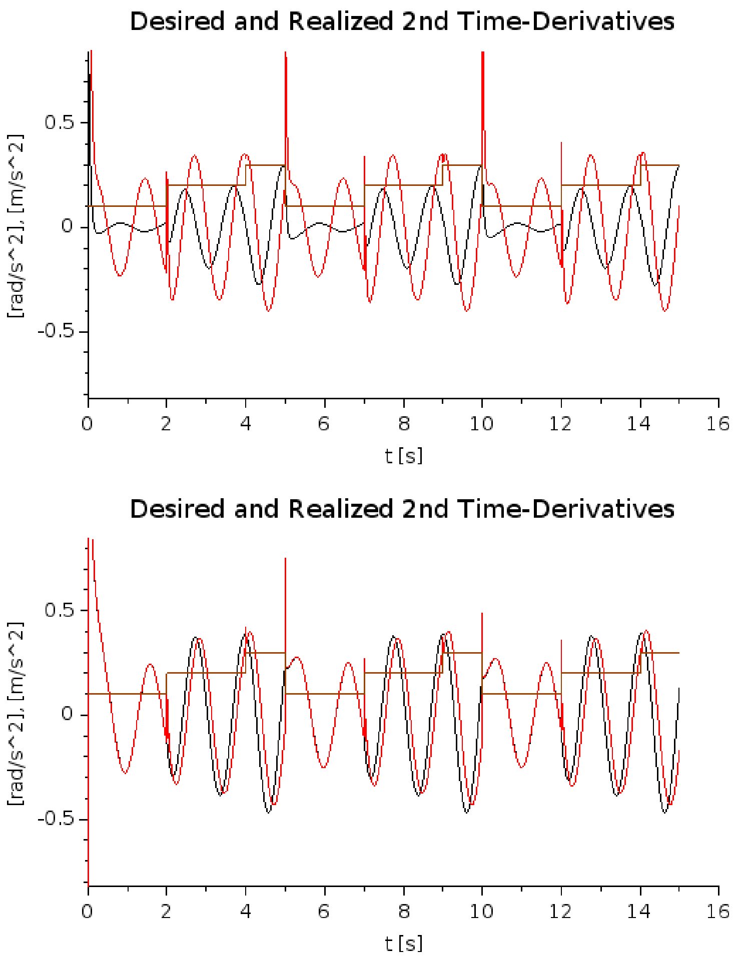

Figure 10
The "desired" $\ddot{q}_1^{Des}$ (black lines) and the realized $\ddot{q}_1$ (red lines) values in the non-adaptive (upper chart) and the adaptive (lower chart) cases; the time slots are indicated by the step function (brown line): increasing values belong to q_1, q_2, and q_3, respectively

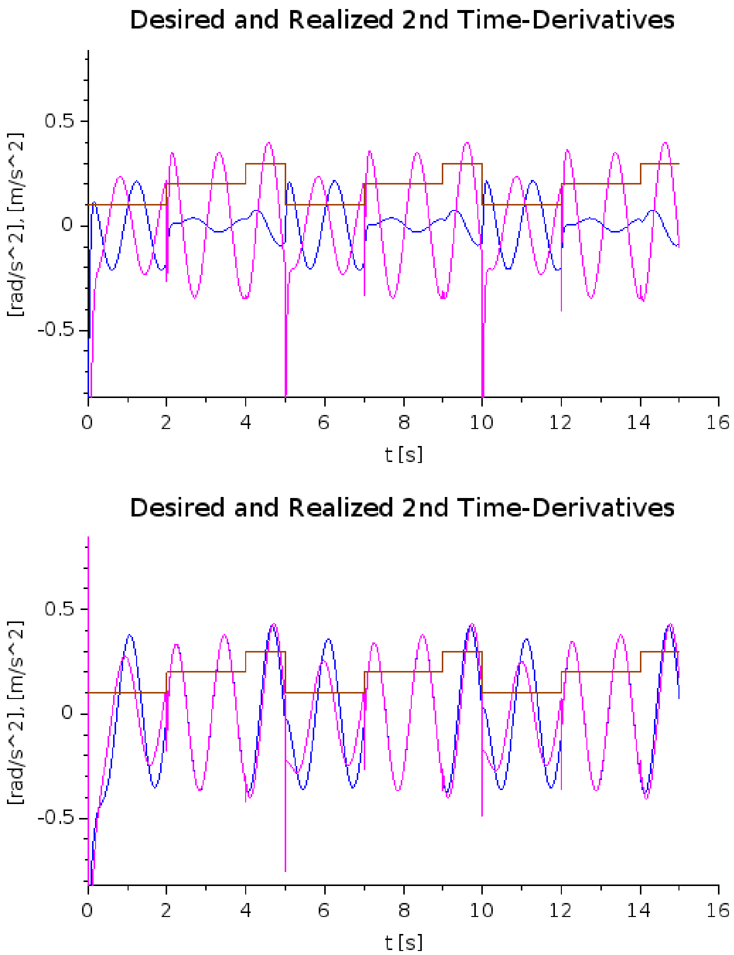

Figure 11
The "desired" $\ddot{q}_2^{Des}$ (blue lines) and the realized $\ddot{q}_2$ (magenta lines) values in the non-adaptive (upper chart) and the adaptive (lower chart) cases; the time slots are indicated by the step function (brown line): increasing values belong to q_1, q_2, and q_3, respectively

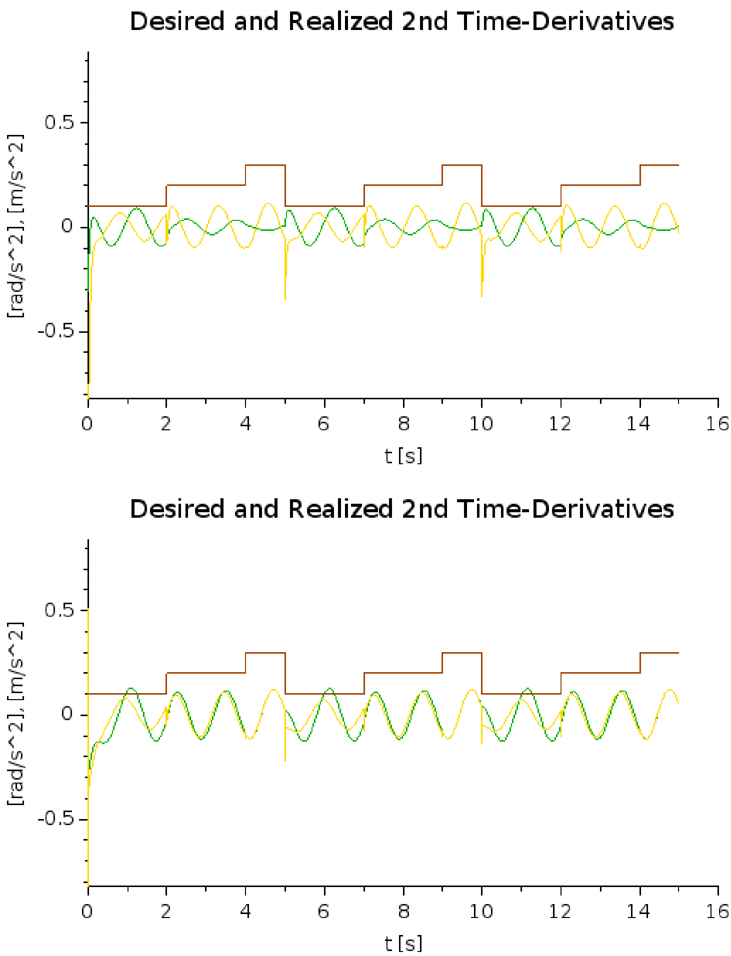

Figure 12
The "desired" $\ddot{q}_3^{Des}$ (green lines) and the realized $\ddot{q}_3$ (ocher lines) values in the non-adaptive (upper chart) and the adaptive (lower chart) cases; the time slots are indicated by the step function (brown line): increasing values belong to q_1, q_2, and q_3, respectively

Evidently, in the appropriate adaptive sessions, the suggested fixed point transformation precisely realized the kinematically prescribed trajectory tracking, for the actually controlled coordinate.

Conclusions

In this paper, a novel fixed point transformation, called *"Stretched Sigmoids Generated Fixed Point Transformation (SSGFPT)"* was suggested, for the realization of *"Adaptive Optimal Control"* for an underactuated Classical Mechanical system, a TORA model.

It was shown that for SISO systems of monotonic increasing response functions of bounded derivatives this controller can realize globally stable operation. This operation is possible because, in this case, the fixed point transformation used for transforming the computation of the necessary control force into the problem of finding the fixed point of a contractive map via iteration, has unbounded basin of attraction. Furthermore, it guarantees very fast convergence, if the actual point is far from the fixed point.

The main point of the optimization considered herein, is the idea of using cost-function free optimization, in which, the necessary compromise between the contradictory prescriptions, is found via time-sharing, realized by rotating time-slots. In contrast to the traditional optimal control, that formally is made complicated, because of the use of cost functionals, the herein applied approach allows simple combination with a non Lyapunov function-based adaptive design.

The numerical simulations well exemplified the operation of the suggested method.

In our future work, we should like to proceed in two separate directions. First, we wish to generalize the SSGFPT method from SISO to multivariable (MIMO) systems. Second, we wish to study the possibilities to include further limiting factors in the computations. Traditionally these factors appear as contributions to the cost functions.

Acknowledgement

This work was supported by the *Research and Innovation Center of Óbuda University*.

At last, but not least, the authors express their thanks to Professor Antal Bejczy with whom they were in professional connection, since the 1980s. At the beginning they met at various conferences. Later, on Professor Bejczy regularly visited Hungary and supported the authors' activities even within "official" frameworks. With this paper the authors wish to give a humble contribution to the commemoration of his great achievements in robotics and control.

References

[1] R.T. Bupp, D.S. Bernstein, and V.T. Coppola. A benchmark problem for nonlinear control design: Problem statement, experiment testbed and passive nonlinear compensation. *Proc. of the American Control Conference, Seattle, US, 1995*, pages 4363–4376, 1995.

[2] M. Jankovic, D. Fontanie, and P.V. Kokotovic. TORA example: Cascade- and passivity based control designs. *IEEE Transaction on Control System Technologies*, 4:292–297, 2006.

[3] P. Baranyi, Z. Petres, P. Várlaki, and P. Michelberger. Observer and control law design to the TORA system via TPDC framework. *WSEAS Transactions on Systems*, 1(5):156–163, 2006.

[4] Authors of a special issue. A special issue presenting 9 papers on controlling the TORA system. *International Journal of Robust and Nonlinear Control*, 8:305–457, 1998.

[5] F. Chee and T. Fernando. *Closed-Loop Control of Blood Glucose*. Springer, 2007.

[6] C. Dalla Man, R.A. Rizza, and C. Cobelli. Meal simulation model of glucose-insulin system. *IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING*, 54(10):1740–1749, 2007.

[7] L. Magni, D.M. Raimondo, L. Bossi, C. Dalla Man, G. De Nicolao, B. Kovatchev, and C. Cobelli. Model Predictive Control of Type 1 Diabetes: An in silico trial. *J Diab Sci Techn*, 1:804–812, 2007.

[8] A.L. Hodgkin and A.F. Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of Physiology*, 117(4):500–544, 1952.

[9] T. Matsumoto. A chaotic attractor from Chua's circuit. *IEEE Transactions on Circuits and Systems*, CAS-31(12):1055–1058, 1984.

[10] R. FitzHugh. Impulses and physiological states in theoretical models of nerve membrane. *Biophys J.*, 1(6):445–466, 1961.

[11] R. Bellman. *Dynamic Programming*. Princeton Univ. Press, Princeton, N. J., 1957.

[12] R.E. Kalman. Contribution to the theory of optimal control. *Boletin Sociedad Matematica Mexicana*, 5(1):102–119, 1960.

[13] Brian D.O. Anderson and John B. Moore. *Optimal Control: Linear Quadratic Methods*. Prentice – Hall International, Inc., A Division of Simon & Schuster, Englewood Cliffs, NJ 07632, 1989.

[14] Tayfun Çimen. State-dependent Riccati equation in nonlinear optimal control synthesis. *In the Proc. of the Special International Conference on Complex Systems: Synergy of Control, Communications and Computing - COSY 2011, Hotel Metropol Resort, Ohrid, Republic of Macedonia, September, 16 – 20, 2011*, pages 321–332, 2011.

[15] J. Richalet, A. Rault, J.L. Testud, and J. Papon. Model predictive heuristic control: Applications to industrial processes. *Automatica*, 14(5):413–428, 1978.

[16] A.M. Lyapunov. *A general task about the stability of motion. (in Russian)*. Ph.D. Thesis, University of Kazan, Tatarstan (Russia), 1892.

[17] A.M. Lyapunov. *Stability of motion*. Academic Press, New-York and London, 1966.

[18] Jean-Jacques E. Slotine and W. Li. *Applied Nonlinear Control*. Prentice Hall International, Inc., Englewood Cliffs, New Jersey, 1991.

[19] C.C. Nguyen, S.S. Antrazi, Zhen-Lei Zhou, and C.E. Campbell Jr. Adaptive control of a Stewart platform-based manipulator. *Journal of Robotic Systems*, 10(5):657–687, 1993.

[20] J. Somló, B. Lantos, and P.T. Cát. *Advanced Robot Control*. Akadémiai Kiadó, Budapest, 2002.

[21] K. Hosseini-Suny, H. Momeni, and F. Janabi-Sharifi. Model Reference Adaptive Control design for a teleoperation system with output prediction. *J Intell Robot Syst*, DOI 10.1007/s10846-010-9400-4:1–21, 2010.

[22] I. Sekaj and V. Veselý. Robust output feedback controller design: Genetic Algorithm approach. *IMA J Math Control Info*, 22(3):257–265, 2005.

[23] J.L. Chen and Wei-Der Chang. Feedback linearization control of a two-link robot using a Multi-Crossover Genetic Algorithm. *Expert Systems with Applications*, 2(2 Part 2):4154–4159, 2009.

[24] S.V. Emelyanov, S.K. Korovin, and L.V. Levantovsky. Higher order sliding regimes in the binary control systems. *Soviet Physics*, 31:291–293, 1986.

[25] V.I. Utkin. *Sliding Modes in Optimization and Control Problems*. Springer Verlag, New York, 1992.

[26] A. Levant. Arbitrary-order sliding modes with finite time convergence. *In Proc. of the 6th IEEE Mediterranean Conference on Control and Systems, June 9-11, 1998, Alghero, Sardinia, Italy*, 1998.

[27] J.K. Tar, J.F. Bitó, L. Nádai, and J.A. Tenreiro Machado. Robust Fixed Point Transformations in adaptive control using local basin of attraction. *Acta Polytechnica Hungarica*, 6(1):21–37, 2009.

[28] J.K. Tar. *Adaptive Control of Smooth Nonlinear Systems Based on Lucid Geometric Interpretation (DSc Dissertation)*. Hungarian Academy of Sciences, Budapest, Hungary, 2012.

[29] Tjalling J. Ypma. Historical development of the Newton-Raphson method. *SIAM Review*, 37(4):531–551, 1995.

[30] C.T. Kelley. *Solving Nonlinear Equations with Newton's Method, no 1 in Fundamentals of Algorithms*. SIAM, 2003.

[31] P. Deuflhard. *Newton Methods for Nonlinear Problems. Affine Invariance and Adaptive Algorithms, Springer Series in Computational Mathematics, Vol. 35*. Springer, Berlin, 2004.

[32] S. Banach. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales (About the Operations in the Abstract Sets and Their Application to Integral Equations). *Fund. Math.*, 3:133–181, 1922.

[33] S. Arimoto, S. Kawamura, and F. Miyazaki. Bettering operation of robots by learning. *Journal of Robotic Systems*, 1(2):123–140, 1984.

[34] S. Gunnarsson and M. Norrlöf. On the design of ILC algorithms using optimization. *Automatica*, 37:2011–2016, 2001.

[35] D.H. Owens and J. Hätönen. Iterative learning control – An optimization paradigm. *Annual Reviews in Control*, 29:57–70, 2005.

[36] Y. Wang, F. Gao, and F.J. Doyle III. Survey on iterative learning control, repetitive control, and run-to-run control. *Journal of Process Control*, 19:1589–1600, 2009.

[37] M. Deniša, A. Ude, and A. Gams. Adaptation of motor primitives to the environment through learning and statistical generalization. *Proc. of the 24th International Workshop on Robotics in Alpe Adria Danube Region (RAAD2015), May 27-29 2015, Bucharest, Romania*, pages 1–8, 2015.

[38] J.K. Tar. *Towards Replacing Lyapunov's "Direct" Method in Adaptive Control of Nonlinear Systems (invited plenary lecture at the Mathematical Methods in Engineering Intl. Symp. (MME), 21-24 October 2010, Coimbra, Portugal), chapter in Mathematical Methods in Engineering (Eds.: N.M. Fonseca Ferreira & J.A. Tenreiro Machado)*. Springer Dordrecht, Heidelberg, New York, London, 2014.

[39] J.K. Tar, L. Nádai, I.J. Rudas, and T.A. Várkonyi. RFPT-based adaptive control stabilized by fuzzy parameter tuning. *In Proc. of the* 9^{th} *European Workshop on Advanced Control and Diagnosis (ACD 2011), Budapest, Hungary*, pages 1–8, 2011.

[40] K. Kósi, J.K. Tar, and I.J. Rudas. Improvement of the stability of RFPT-based adaptive controllers by observing "precursor oscillations". *In Proc. of the* 9^{th} *IEEE Intl. Conf. on Computational Cybernetics, Tihany, Hungary*, pages 267–272, 2013.

[41] A. Dineva, J.K. Tar, and A.R. Várkonyi-Kóczy. Novel generation of Fixed Point Transformation for the adaptive control of a nonlinear neuron model. *In proc. of the IEEE International Conference on Systems, Man, and Cybernetics, October 10-13, 2015, Hong Kong (SMC 2015)*, pages 987–992, 2015.

[42] A. Dineva, J.K. Tar, A.R. Várkonyi-Kóczy, and V. Piuri. Generalization of a sigmoid generated fixed point transformation from SISO to MIMO systems. *Accepted for publication at the IEEE 19th International Conference on Intelligent Engineering Systems, September 3-5, 2015, Bratislava, Slovakia (INES 2015)*, page paper no. 3, 2015.

[43] A. Dineva, J.K. Tar, A.R. Várkonyi-Kóczy, and V. Piuri. Replacement of parameter tuning with simple calculation in adaptive control using "Sigmoid Generated Fixed Point Transformation". *Accepted for publication at the 13th IEEE International Symposium on Intelligent Systems and Informatics, September 17-19, Subotica, Serbia (SISY 2015)*, page paper no. 3, 2015.

[44] K. Kósi, J.F. Bitó, and J.K. Tar. On the effects of strong asymmetries on the adaptive controllers based on robust fixed point transformations. *In Proc. of*

the IEEE 10^{th} *Jubilee Intl. Symp. on Intelligent Systems and Informatics (SISY 2012), Subotica, Serbia*, pages 259–264, 2012.

[45] K. Kósi, A. Dineva, and J.K. Tar. Increased cycle time achieved by fractional derivatives in the adaptive control of the Brusselator model. *In Proc. of the* 11^{th} *IEEE Intl. Symp. on Applied Machine Intelligence and Informatics (SAMI), Herl'any, Slovakia*, pages 65–70, 2013.

[46] B. Kurtán. *Investigation and Adaptive Synchronization of Physiological Neuron Models via Numerical Simulations (MSc Thesis, Supervisor: J.K. Tar)*. John von Neumann Faculty of Informatics, Óbuda University, Budapest, Hungary, 2014.

[47] J.F. Bitó and J.K. Tar. Fixed point transformation-based adaptive control of the Hodgkin-Huxley neuron. *Proc. of the Workshop on Information Technology and Bionics – Symposium in Memory of Tamás Roska, 23-24 June, 2015, Pázmány University* e*Press, Budapest, Hungary*, pages 69–72, 2015.

[48] L. Nádai and I. Felde. Adaptive control by using time-sharing and Fixed Point Transformation. *In proc. of the IEEE International Conference on Systems, Man, and Cybernetics, October 10-13, 2015, Hong Kong (SMC 2015)*, pages 2492–2497, 2015.

[49] Gy. Eigner, J.K. Tar, and L. Kovács. Adaptive control solution for T1DM control. *In: Proc. of the 10th IEEE Jubilee International Symposium on Applied Computational Intellience and Informatics (SACI 2015), from May 21 to 23 2015, Timişoara, Romania*, pages 215–220, 2015.

[50] Gy. Eigner, P. Horváth, J.K. Tar, I.J. Rudas, and L. Kovács. Application of Robust Fixed Point control in case of T1DM. *In proc. of the IEEE International Conference on Systems, Man, and Cybernetics, October 10-13, 2015, Hong Kong (SMC 2015)*, pages 2459–2463, 2015.

[51] K. Kósi. *Further development and novel applications of the Robust Fixed Point Transformation-based adaptive control (PhD Thesis, supervisor: J.K. Tar)*. Óbuda University, Budapest, Hungary, 2015.

[52] K. Kósi, Sz. Hajdu, J.F. Bitó, and J.K. Tar. Chaos formation and reduction in Robust Fixed Point Transformations based adaptive control. *In Proc. of the 4th IEEE Intl. Conf. on Nonlinear Science and Complexity (NSC 2012), Budapest, Hungary*, pages 211–216, 2012.

[53] K. Kósi, Á. Breier, and J.K. Tar. Chaos patterns in a 3 degree of freedom control with robust fixed point transformation. *In Proc. of the* 13^{th} *IEEE Intl. Symp. on Computational Intelligence and Informatics, Budapest, Hungary*, pages 1–5, 2012.