Data-driven Model-Free Adaptive Control Tuned by Virtual Reference Feedback Tuning

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Abstract: This paper proposes a new tuning approach, by which, all parameters of a data-driven Model-Free Adaptive Control (MFAC) algorithm are automatically determined using a nonlinear Virtual Reference Feedback Tuning (VRFT) algorithm. The approach is referred to as mixed MFAC-VFRT control and it leads to mixed MFAC-VFRT algorithms. An advantage of mixed MFAC-VFRT control, is that it combines systematically, the features of VRFT (it computes the controller parameters using only the input/output data) with those of MFAC. This is especially illustrated by comparison with the classical MFAC algorithms, the initial values of the parameters, which are obtained through a process that involves solving an optimization problem. The application that validates the mixed MFAC-VFRT algorithms, by experiment, is a nonlinear twin rotor aerodynamic system laboratory equipment position control system, that represents a tribute, to Prof. Antal (Tony) K. Bejczy for his excellent results in space robotics, robot dynamics and control, haptics and force perception/control.

Keywords: Model-Free Adaptive Control; twin rotor aerodynamic system; optimal control; state-space model; Virtual Reference Feedback Tuning

1 Introduction

Virtual Reference Feedback Tuning (VRFT) is a technique used for data-driven controllers. VRFT was first proposed and applied in [1] to Single Input-Single Output (SISO) systems, then in [2] Multi Input-Multi Output (MIMO) systems and next extended in [3, 4] a nonlinear version. The main process of VRFT consists of collecting the input/output (I/O) data from an unknown open-loop process, and with this data, computes the controller parameters. A disadvantage of this technique is that it does not guarantee the closed loop control system (CS) stability.

As presented in [5, 6], the main features of Model-Free Adaptive Control (MFAC) is that MFAC algorithms make use of only the online I/O data of the process, and they ensure CS stability through reset conditions related to a so-called Pseudo-Partial-Derivatives (PPD) matrix.

Using the complementary features of MFAC and VRFT, this paper proposes a mixed MFAC-VFRT control approach. This mixed algorithm is also successfully applied in [7] to a class of nonlinear MIMO systems. The approach aims to control the azimuth and pitch motions of the Twin Rotor Aerodynamic System (TRAS), i.e., a representative process for nonlinear robotics, space and automotive applications [8]-[12] with focus on the seminal contributions of Prof. Antal (Tony) K. Bejczy, to whose memory, this paper is dedicated.

As proven in [13] for TRAS, the MFAC algorithms behave practically, like classical integral controllers, because the PPD matrix is almost constant, and this motivates the need for combination with other data-driven techniques. The mixed MFAC-VFRT control approach is time saving in finding the optimal parameters of the classical MFAC algorithm, which has five parameters in the SISO scenario and eight parameters in the MIMO scenario, for the TRAS laboratory equipment considered in this paper. This is especially important, as a basis for other combinations of data-driven control approaches [14, 15].

The paper is organized as follows. Section 2 describes the TRAS laboratory equipment. An overview on MFAC and nonlinear VRFT is presented in Section 3. The MFAC-VFRT control approach is shown in Section 4. The experimental validation is done in Section 5 and conclusions are outlined in the final section.

2 Twin Rotor Aerodynamic System

The nonlinear state-space model that describes the MIMO TRAS process is [16]:

$$\begin{aligned}
&\dot{\Omega}_h = [l_t F_h(\omega_t)\cos\alpha_v + \Omega_h f_h + u_2 k_{vh}] / J_h, \\
&\dot{\Omega}_v = \{l_m F_v(\omega_m) + \Omega_v f_v + g[(A-B)\cos\alpha_v - C\sin\alpha_v] \\
&-(\Omega_h^2/2)(A+B+C)\sin 2\alpha_v\} / J_v, \\
&\dot{\alpha}_h = \Omega_h,\ \dot{\alpha}_v = \Omega_v,\ \dot{\omega}_h = (u_1 - \omega_h / k_{Hh}) / I_h,\ \dot{\omega}_v = (u_2 - \omega_v / k_{Hv}) / I_v, \\
&y_1 = \alpha_h, y_2 = \alpha_v,
\end{aligned} \tag{1}$$

where: u_1 [%] – the first control input, i.e., the PWM duty cycle of the horizontal (main) direct current (DC) motor, u_2 [%] – the second control input, i.e., the

PWM duty cycle of the vertical (tail) DC motor, $\alpha_h\,[\text{rad}] = y_1$ – the first process output, i.e., the azimuth (horizontal) position of the beam that supports the main and the tail rotor, $\alpha_v\,[\text{rad}] = y_2$ – the second process output, i.e., the pitch (vertical) position of the beam. The linearization of (1) at the equilibrium point leads to the linearized state-space model of the process, which consists of the third to eighth equations plus the first two equations replaced by [17, 18]:

$$\dot{\Omega}_h = a_{14}\alpha_v + a_{15}\omega_h,\ \dot{\Omega}_v = a_{34}\alpha_v + a_{36}\omega_v, \tag{2}$$

where all variables are expressed as deviations with respect to the equilibrium point.

The typical control objective for TRAS is to ensure the regulation and tracking for vertical and horizontal motions, i.e., to control the azimuth and the pitch. This paper considers a MIMO CS that is decomposed into two SISO CSs, namely the azimuth control loop and the pitch control loop. Although the theory will be presented as follows, in the general MIMO case, the experimental results will be given in Section 5 for both SISO CSs.

3 Overview on MFAC and Nonlinear VRFT

3.1 MFAC

MFAC is developed using the MIMO nonlinear discrete-time process model:

$$\mathbf{y}(k+1) = \mathbf{f}(\mathbf{y}(k),...,\mathbf{y}(k-n_y),\mathbf{u}(k),...,\mathbf{u}(k-n_u)), \tag{3}$$

where $\mathbf{y}(k) = [y_1(k) \quad y_2(k)]^T \in \mathbf{R}^{2\times 1}$ is the controlled output vector, $\mathbf{u}(k) = [u_1(k) \quad u_2(k)]^T \in \mathbf{R}^{2\times 1}$ is the control input vector, T stands for matrix transposition, n_y and n_u are the unknown process orders and $\mathbf{f}$ is an unknown nonlinear vector-valued function, $\mathbf{f}: \mathbf{R}^{2(n_y+n_u+2)} \to \mathbf{R}^2$. The partial derivatives of $\mathbf{f}$ with respect to the elements of the vector $\mathbf{u}(k)$ (i.e., the control inputs) are assumed to be continuous.

Since the Compact Form Dynamic Linearization (CFDL) is the most popular version of MFAC [5], this paper treats only the CFDL version. The results can be extended to other versions as well. According to [5] the PPD matrix $\mathbf{\Phi}(k)$ exists such that (3) can be transformed into the following CFDL-MFAC data model:

$$\Delta\mathbf{y}(k+1) = \mathbf{\Phi}(k)\Delta\mathbf{u}(k), \tag{4}$$

where $\mathbf{\Phi}(k)=[\phi_{ij}(k)]_{i,j\in\{1,2\}}$, $\|\mathbf{\Phi}(k)\|\le b$. These conditions concerning $\mathbf{\Phi}(k)$ are met only if the model in (3) is assumed Lipschitz, i.e., $\|\Delta\mathbf{y}(k+1)\|\le b\|\Delta\mathbf{u}(k)\|$ for each fixed discrete time moment k, and $\|\Delta\mathbf{u}(k)\|\neq 0$, with $\Delta\mathbf{y}(k+1)=\mathbf{y}(k+1)-\mathbf{y}(k)$, $\Delta\mathbf{u}(k)=\mathbf{u}(k)-\mathbf{u}(k-1)$ and $b=\text{const}>0$.

The MFAC objective is to solve the optimization problem [5]:

$$\begin{aligned}&\mathbf{u}^*(k)=\arg\min_{\mathbf{u}(k)} J_{MFAC}(\mathbf{u}(k)),\\&J_{MFAC}(\mathbf{u}(k))=\|\mathbf{y}^*(k+1)-\mathbf{y}(k+1)\|^2+\lambda\|\Delta\mathbf{u}(k)\|^2,\end{aligned}\tag{5}$$

where $\mathbf{y}^*(k+1)=[y_1^*(k+1)\quad y_2^*(k+1)]^T$ is the tracking reference input vector and $\lambda\ge 0$ is a weighting parameter. The estimate of $\mathbf{\Phi}(k)$ is computed using the I/O data from the process, this matrix should be diagonally dominant and bounded:

$$|\phi_{ij}(k)|\le b_1,\ b_2\le|\phi_{ii}(k)|\le a\,b_2, i,j\in\{1,2\}, i\neq j, a\ge 1, b_2>b_1(2a+1),\tag{6}$$

where the signs of all elements of $\mathbf{\Phi}(k)$ should remain unchanged.

The estimate $\hat{\mathbf{\Phi}}(k)$ of the PPD matrix $\mathbf{\Phi}(k)$ is:

$$\hat{\mathbf{\Phi}}(k)=\hat{\mathbf{\Phi}}(k-1)+\frac{\eta[\Delta\mathbf{y}(k)-\hat{\mathbf{\Phi}}(k-1)\Delta\mathbf{u}(k-1)]\Delta\mathbf{u}^T(k-1)}{\mu+\|\Delta\mathbf{u}(k-1)\|^2},\tag{7}$$

where $0<\eta<1$ is a step size constant and $\mu>0$ is another weighting factor parameter, different to optimal control. The resetting conditions are:

$$\begin{aligned}&\hat{\phi}_{ii}(k)=\hat{\phi}_{ii}(1),\text{ if } |\hat{\phi}_{ii}(k)|<b_2 \text{ or } |\hat{\phi}_{ii}(k)|>a\,b_2 \text{ or } \operatorname{sgn}(\hat{\phi}_{ii}(k))\neq\operatorname{sgn}(\hat{\phi}_{ii}(1)),\\&\hat{\phi}_{ij}(k)=\hat{\phi}_{ij}(1),\text{ if } |\hat{\phi}_{ij}(k)|>b_1 \text{ or } \operatorname{sgn}(\hat{\phi}_{ij}(k))\neq\operatorname{sgn}(\hat{\phi}_{ij}(1)),\ i\neq j,\end{aligned}\tag{8}$$

where $\hat{\phi}_{ij}(1)$ is the initial value of $\hat{\phi}_{ij}(k)$, $i\in\{1,2\}$, $j\in\{1,2\}$. According to [5], the substitution of $\mathbf{y}(k+1)=\mathbf{y}(k)+\mathbf{\Phi}(k)\Delta\mathbf{u}(k)$ into (5) leads to the control law specific to MFAC algorithms:

$$\mathbf{u}(k)=\mathbf{u}(k-1)+\frac{\rho\hat{\mathbf{\Phi}}^T(k)[\mathbf{y}^*(k+1)-\mathbf{y}(k)]}{\lambda+\|\hat{\mathbf{\Phi}}(k)\|^2},\tag{9}$$

where $\rho>0$ is another step size constant. Finding the parameters $\hat{\mathbf{\Phi}}(1)$, ρ, η, λ, μ of the MFAC algorithm is a difficult task, without a model of the controlled process and guidelines for appropriate selection, which do not exist to the best of authors' knowledge. This procedure involving a process model is usually an optimization problem, which is solved for a specified control scenario as illustrated in [17, 18]. However, this defies the purpose of MFAC and prevents it from being a truly model-free approach. The parameters of MFAC are obtained in a nonlinear VRFT framework that will be introduced in Section 4.

3.2 Nonlinear VRFT

Nonlinear VRFT uses a linear or a nonlinear reference model, which ultimately must be tracked by the closed-loop CS. Nonlinear VRFT uses only a single open-loop experiment, where a rich spectrum frequency signal is applied as input to the stable nonlinear process, then the I/O signals are collected, and then used to compute the controller parameters [3, 4, 19].

The model reference objective function (o.f.) used in nonlinear VRFT is [19]:

$$J_{MR}(\boldsymbol{\theta}) = \sum_{k=1}^{N} \left\| \mathbf{y}_{\boldsymbol{\theta}}(k) - \mathbf{y}^{d}(k) \right\|^{2}, \tag{10}$$

where $\mathbf{y}_{\boldsymbol{\theta}}(k+1) = \mathbf{f}(\mathbf{y}(k),...,\mathbf{y}(k-n_y),\mathbf{u}_{\boldsymbol{\theta}}(k),...,\mathbf{u}_{\boldsymbol{\theta}}(k-n_u))$ is the nonlinear process output vector, $\mathbf{u}_{\boldsymbol{\theta}}(k) = C_{\boldsymbol{\theta}}(\boldsymbol{\theta},\mathbf{u}(k-1),\ldots,\mathbf{u}(k-n_{uc}),\mathbf{e}(k),\ldots,\mathbf{e}(k-n_{ec}))$ (in shorthand notation expressed as $\mathbf{u}_{\boldsymbol{\theta}}(k) = C_{\boldsymbol{\theta}}(\boldsymbol{\theta},\mathbf{u}(k-1),\mathbf{e}(k))$) is the nonlinear controller output vector, with uc and ec – the known orders of the fixed structure controller parameterized by the vector $\boldsymbol{\theta}$, $\mathbf{e}(k) = \mathbf{r}(k) - \mathbf{y}_{\boldsymbol{\theta}}(k)$ is the tracking error, $\mathbf{r}(k)$ is the reference input vector applied to the closed-loop CS, $\mathbf{y}^{d}(k) = \mathbf{m}(\mathbf{y}^{d}(k-1),\ldots,\mathbf{y}^{d}(k-n_{\mathbf{y}m}),\mathbf{r}(k-1),\ldots,\mathbf{r}(k-n_{\mathbf{r}m}))$ is the output of the user-selected nonlinear reference model $\mathbf{m}$ of orders $\mathbf{y}m$ and $\mathbf{r}m$ accepting that the input is set as $\mathbf{r}(k)$. It is assumed that $\mathbf{m}$ is non-singular.

VRFT assumes that an I/O pair of data $\{\mathbf{u}(k),\mathbf{y}(k)\}$, $k = 0...N$, are available from the open-loop stable process. Then a virtual reference input vector $\bar{\mathbf{r}}(k)$ is calculated as $\bar{\mathbf{r}}(k) = \mathbf{m}^{-1}(\mathbf{y}(k))$, such that the reference model output and the closed-loop CS output have similar trajectories. By enforcing the notation of $\mathbf{m}^{-1}(\mathbf{y}(k))$ results in $\bar{\mathbf{r}}(k)$, which set as input to $\mathbf{m}$ and gives $\mathbf{y}(k)$. The virtual reference tracking error is then $\bar{\mathbf{e}}(k) = \bar{\mathbf{r}}(k) - \mathbf{y}(k)$. The controller which achieves $\mathbf{u}(k)$ if $\bar{\mathbf{e}}(k)$ is applied to its input is the one achieving reference model tracking. The parameters of this controller are calculated by minimizing the o.f. [19]:

$$J_{VRFT}(\boldsymbol{\theta}) = \frac{1}{N} \sum_{k=1}^{N} \left\| \mathbf{C}_{\boldsymbol{\theta}}(\boldsymbol{\theta}, \bar{\mathbf{e}}(k)) - \mathbf{u}(k) \right\|^{2}, \tag{11}$$

According to [19], in MIMO VRFT there is no need for any time-varying filter to make $J_{MR}(\boldsymbol{\theta})$ and $J_{VR}(\boldsymbol{\theta})$ approximately equal, as is usually the case in classical VRFT. The two o.f.s can be made approximately equal for a rich parameterization of the controller, which can be, for example, a neural network [19, 20]. The same nonlinear VRFT theory can be used for SISO CS design as a particular case.

4 Mixed MFAC-VRFT Control Approach

This section shows that VRFT can be used to find the parameters of MFAC algorithms. First, it will be shown that a general MFAC algorithm comprised of the estimation mechanism (7) and the control law (9) can be expressed as both a state-space nonlinear model and an I/O nonlinear recurrence. Let the nonlinear state-space model of the MFAC controller be:

$$\begin{aligned}
&\mathbf{u}(k)=\mathbf{u}(k-1)+\frac{\rho\hat{\mathbf{\Phi}}^T(k)[\mathbf{y}^*(k+1)-\mathbf{y}(k)]}{\lambda+\|\hat{\mathbf{\Phi}}(k)\|^2},\\
&\hat{\mathbf{\Phi}}(k)=\hat{\mathbf{\Phi}}(k-1)+\frac{\eta[\Delta\mathbf{y}(k)-\hat{\mathbf{\Phi}}(k-1)(\mathbf{u}(k-1)-\mathbf{u}(k-2))](\mathbf{u}^T(k-1)-\mathbf{u}^T(k-2))}{\mu+\|\mathbf{u}(k-1)-\mathbf{u}(k-2)\|^2},
\end{aligned} \tag{12}$$

equivalent to:

$$\begin{aligned}
&\mathbf{u}(k)=\mathbf{g}(\hat{\mathbf{\Phi}}(k),\mathbf{u}(k-1),\mathbf{y}^*(k+1),\mathbf{y}(k),\boldsymbol{\theta}),\\
&\hat{\mathbf{\Phi}}(k)=\mathbf{h}(\hat{\mathbf{\Phi}}(k-1),\mathbf{u}(k-1),\mathbf{u}(k-2),\mathbf{y}(k),\mathbf{y}(k-1),\boldsymbol{\theta}),
\end{aligned} \tag{13}$$

where $\mathbf{g},\mathbf{h}\in\mathbf{R}^{2\times1}$ are nonlinear functions of their arguments. By introducing the additional state vector $\mathbf{z}(k)=\mathbf{u}(k-1)$, it can be shown that a state-space mathematical model is in the form $\boldsymbol{\chi}(k)=\mathbf{F}(\boldsymbol{\chi}(k-1),\mathbf{U}(k),\boldsymbol{\theta})$, where the state vector is $\boldsymbol{\chi}(k)=[\mathbf{u}(k)^T\ \ \mathbf{z}(k)^T\ \ \hat{\mathbf{\Phi}}(k)^T]^T$, the input vector is $\mathbf{U}(k)=[\mathbf{y}^*(k+1)^T\ \ \mathbf{y}(k)^T\ \ \mathbf{y}(k-1)^T]^T$ and the parameter vector is $\boldsymbol{\theta}=[\rho\ \ \eta\ \ \lambda\ \ \mu]^T$, which is considered as an additional input vector (i.e., disturbance vector).

Using the above notations and replacing $\hat{\mathbf{\Phi}}(k)$ from the first equation in (13) with the second one, the following state-space form of the MIMO MFAC algorithm is obtained:

$$\begin{aligned}
&\mathbf{u}(k)=\mathbf{g}(\hat{\mathbf{\Phi}}(k-1),\mathbf{u}(k-1),\mathbf{z}(k-1),\mathbf{y}(k),\mathbf{y}(k-1),\mathbf{y}^*(k+1),\boldsymbol{\theta}),\\
&\mathbf{z}(k)=\mathbf{u}(k-1),\\
&\hat{\mathbf{\Phi}}(k)=\mathbf{h}(\hat{\mathbf{\Phi}}(k-1),\mathbf{z}(k),\mathbf{z}(k-1),\mathbf{y}(k),\mathbf{y}(k-1),\boldsymbol{\theta}).
\end{aligned} \tag{14}$$

Starting with the initial conditions $\hat{\mathbf{\Phi}}(1),\mathbf{u}(1),\mathbf{z}(1)=\mathbf{u}(0)$ applied to the nonlinear state-space model given in (14), the control input vector $\mathbf{u}(k)$ is expressed recurrently:

$$
\begin{aligned}
&\hat{\mathbf{\Phi}}(2)=\mathbf{h}(\hat{\mathbf{\Phi}}(1),\mathbf{u}(1),\mathbf{u}(0),\mathbf{y}(2),\mathbf{y}(1),\boldsymbol{\theta}),\\
&\mathbf{u}(2)=\mathbf{g}(\hat{\mathbf{\Phi}}(1),\mathbf{u}(1),\mathbf{u}(0),\mathbf{y}(2),\mathbf{y}(1),\mathbf{y}^*(3),\boldsymbol{\theta}),\\
&\mathbf{u}(3)=\mathbf{g}(\hat{\mathbf{\Phi}}(2),\mathbf{u}(2),\mathbf{u}(1),\mathbf{y}(3),\mathbf{y}(2),\mathbf{y}^*(4),\boldsymbol{\theta})=\mathbf{g}(\mathbf{h}(\hat{\mathbf{\Phi}}(1),\mathbf{u}(1),\mathbf{u}(0),\mathbf{y}(2),\\
&\quad \mathbf{y}(1),\boldsymbol{\theta}),\mathbf{g}(\hat{\mathbf{\Phi}}(1),\mathbf{u}(1),\mathbf{u}(0),\mathbf{y}(2),\mathbf{y}(1),\mathbf{y}^*(3),\boldsymbol{\theta}),\mathbf{u}(1),\mathbf{y}(3),\mathbf{y}(2),\mathbf{y}^*(4),\boldsymbol{\theta})\\
&\quad =\mathbf{g}(\hat{\mathbf{\Phi}}(1),\mathbf{u}(1),\mathbf{u}(0),\mathbf{y}(3),\mathbf{y}(2),\mathbf{y}(1),\mathbf{y}^*(3),\mathbf{y}^*(4),\boldsymbol{\theta}),\\
&\ldots\\
&\mathbf{u}(k)=\mathbf{g}(\hat{\mathbf{\Phi}}(1),\mathbf{u}(1),\mathbf{u}(0),\mathbf{y}(k),\mathbf{y}(k-1),\ldots,\mathbf{y}(2),\mathbf{y}(1),\mathbf{y}^*(k+1),\mathbf{y}^*(k),...,\\
&\quad \mathbf{y}^*(4),\mathbf{y}^*(3),\boldsymbol{\theta})=\mathbf{g}(\hat{\mathbf{\Phi}}(1),\mathbf{u}(1),\mathbf{u}(0),\mathbf{y}^*(k+1)-\mathbf{y}(k),\mathbf{y}^*(k)-\mathbf{y}(k-1),\ldots,\\
&\quad \mathbf{y}^*(3)-\mathbf{y}(2),\mathbf{y}(1),\boldsymbol{\theta}).
\end{aligned}
\tag{15}
$$

If we denote $\mathbf{e}(k)=\mathbf{y}^*(k+1)-\mathbf{y}(k)$ then $\mathbf{u}(k)$ in (15) can be considered to emerge from an input-output nonlinear recurrent description of the form $\mathbf{u}_{\boldsymbol{\theta}_e}(k)=C_{\boldsymbol{\theta}_e}(\boldsymbol{\theta}_e,\mathbf{u}(k-1),\ldots,\mathbf{u}(k-n_{uc}),\mathbf{e}(k),\ldots,\mathbf{e}(k-n_{ec}))$, with $\boldsymbol{\theta}_e=\{\hat{\mathbf{\Phi}}(1),\boldsymbol{\theta}^T\}$. If $\mathbf{r}(k)$ specific to VRFT is considered equivalent to $\mathbf{y}^*(k+1)$ in MFAC, then the MFAC controller structure can be considered in a closed-loop CS. Figure 1 shows the CS structure with MFAC-VRFT algorithm.

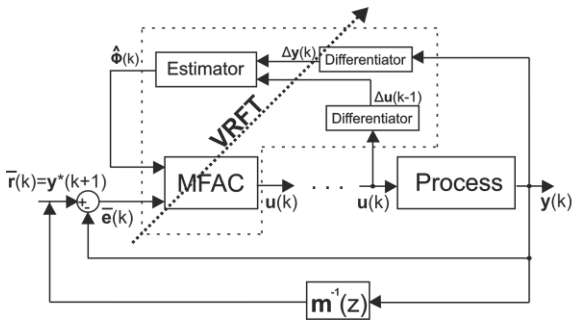

Figure 1
CS structure with mixed MFAC-VRFT algorithm [7]

Choosing the reference model $\mathbf{m}=\mathbf{I}_2$ in the nonlinear VRFT design is equivalent to trying to minimize $J_{MR}(\boldsymbol{\theta})=\sum_{k=1}^{N}\left\|\mathbf{y}_{\boldsymbol{\theta}}(k)-\mathbf{y}^*(k)\right\|^2$, which is the batch-wise version of the adaptive one-step ahead MFAC o.f. J_{MFAC} in (5) with $\lambda=0$. However, no causal controller that can achieve $\mathbf{m}=\mathbf{I}_2$ exists in practice. Choosing therefore $\mathbf{m}\neq\mathbf{I}_2$ in VRFT is equivalent to $\lambda\neq 0$ in MFAC. The parameter λ is crucial since it impacts the MFAC stability in the sense that an increased λ improves stability which simply means adding more weight to the control input increment. In terms of VRFT, this means choosing a reference model $\mathbf{m}$ with lower bandwidth which increases the CS response time but also increases the overall CS robustness. Therefore, the proposed mixed MFAC-VRFT control approach translates the design of MFAC algorithm parameters (such as $\hat{\mathbf{\Phi}}(1)$ and $\mathbf{\theta}=[\rho\ \ \eta\ \ \lambda\ \ \mu]^T$) into easier to comprehend closed-loop CS characteristics described by the reference model $\mathbf{m}$.

5 SISO Experimental Validation

Two tuning strategies are described in this section in order to validate the mixed MFAC-VRFT control approach:

- an indirect one, in which the VRFT framework is used and the o.f. in (11) is minimized, this is the mixed MFAC-VRFT control approach
- a direct tuning approach, in which a process model is used and a metaheuristics Gravitational Search Algorithm (GSA) optimizer [21]-[23] is used to minimize the o.f.s:

$$\begin{aligned} &\mathbf{\tau}^* = \arg\min_{\mathbf{\tau}} J(\mathbf{\tau}),\ \ J_\varepsilon^a(\mathbf{\tau}) = \frac{1}{N}\sum_{k=1}^{N}((\mathbf{y}_1^*(k,\mathbf{\tau}) - \mathbf{y}_1(k,\mathbf{\tau}))^2), \\ &\mathbf{\tau}^* = \arg\min_{\mathbf{\tau}} J(\mathbf{\tau}),\ \ J_\varepsilon^p(\mathbf{\tau}) = \frac{1}{N}\sum_{k=1}^{N}((\mathbf{y}_2^*(k,\mathbf{\tau}) - \mathbf{y}_2(k,\mathbf{\tau}))^2), \end{aligned} \quad (16)$$

where $\mathbf{\tau}^*$ is the optimal parameter vector of VRFT-MFAC and MFAC algorithms, the expression of the parameter vector is $\mathbf{\tau}=[\hat{\Phi}_a(1)\ \rho_a\ \ \eta_a\ \ \lambda_a\ \ \mu_a]^T$ for azimuth control and $\mathbf{\tau}=[\hat{\Phi}_p(1)\ \rho_p\ \ \eta_p\ \ \lambda_p\ \ \mu_p]^T$ for pitch control, a indicates the azimuth control and p indicates the pitch control. Other optimization problems with adequate o.f.s used as performance indices can be used as well [24]-[26].

The bounds in (6) are set as $b_2 = \hat{\Phi}(1)/2$ and ab_2, where $a=3$. This section will investigate if the performance of CS with mixed VRFT-MFAC algorithm is similar to the performance of CS with MFAC algorithm. The CS performance is assessed through ten experimental trials of the o.f.s J_ε^a and J_ε^p. The averages and variances of these o.f.s. are next taken for the sake of improved measurement of CS and algorithm performance to avoid random disturbances.

The experiments have shown that the performance of CS with mixed VRFT-MFAC algorithm depends on the initial signals applied to the open-loop experiment and also on the reference model $\mathbf{m}$, which, according to [1]-[4] must be chosen such that the closed-loop CS signal should be capable to track the reference model.

Extensive work shows that the choice of the reference model ensuring an overall stable CS is rather restrictive.

The MFAC algosithms are designed using the transfer function matrix:

$$m_a(z) = \frac{0.00079z^{-1} + 0.00079z^{-2}}{1 - 1.981z^{-1} + 0.982z^{-2}} \tag{17}$$

for azimuth control, and:

$$m_p(z) = \frac{0.00176z^{-1} + 0.00172z^{-2}}{1 - 1.938z^{-1} + 0.941z^{-2}} \tag{18}$$

for pitch control. VRFT is next applied to compute the controllers initial parameters using a GSA that minimizes the o.f. in (11). These parameters are: $\hat{\Phi}(1) = 513$, bounded by $\hat{\Phi}(1) \in (256.5,\ 769.5)$, $\rho = 7$, $\eta = 0.0076$, $\lambda = 704$, and $\mu = 993.05$ for azimuth control and $\hat{\Phi}(1) = 4.22$, bounded by $\hat{\Phi}(1) \in (2.11,\ 6.34)$, $\rho = 0.18$, $\eta = 0.0039$, $\lambda = 4.43$, and $\mu = 999.85$ for pitch control.

The initial parameters of the MFAC algorithms obtained by a GSA that minimizes the o.f. in (17) are: $\hat{\Phi}(1) = 110$, bounded by $\hat{\Phi}(1) \in (55,\ 165)$, $\rho = 1.55$, $\eta = 0.1$, $\lambda = 3.65$, and $\mu = 0.89$ for azimuth control, and $\hat{\Phi}(1) = 160$, bounded by $\hat{\Phi}(1) \in (80,\ 240)$, $\rho = 5.35$, $\eta = 0.31$, $\lambda = 6.21$, and $\mu = 0.54$ for pitch control.

Table 1 gives the averages and the variances of J_ε. The CS responses as control inputs and controlled outputs versus time are presented in Figure 2 for the azimuth SISO control loop and in Figure 3 for the pitch SISO control loop. Figures 2 and 3 also illustrate the tracking reference inputs, which can be slightly different for other applications [27]-[31].

Table 1

The values of the o.f.s

	Mixed VRFT-MFAC	MFAC
Average of J_ε^a	0.004	0.0036
Variance of J_ε^a	$1.8990 \cdot 10^{-7}$	$6.5343 \cdot 10^{-7}$
Average of J_ε^p	0.0034	0.0036
Variance of J_ε^p	$1.9739 \cdot 10^{-8}$	$4.5406 \cdot 10^{-9}$

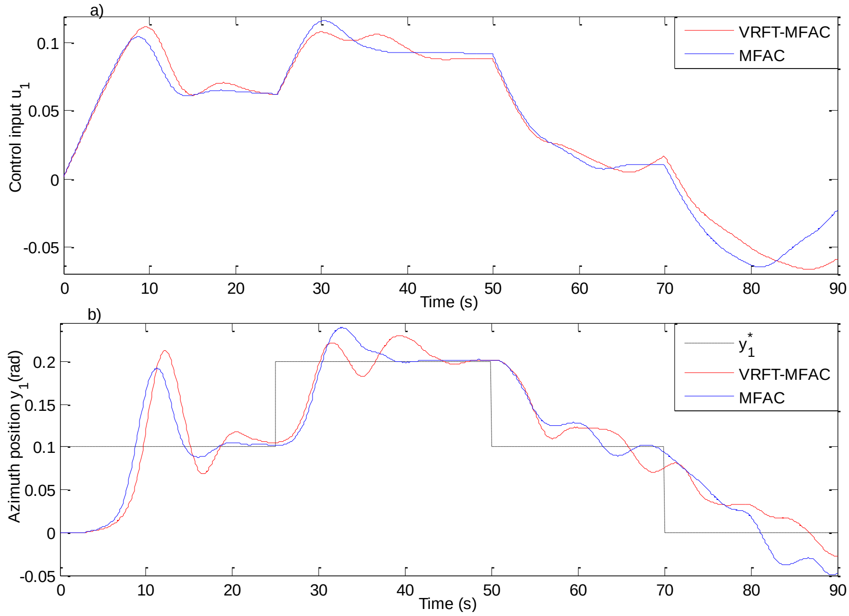

Figure 2
Experimental results related to SISO azimuth control: a) u_1 versus time, b) y_1 and y_1^* versus time

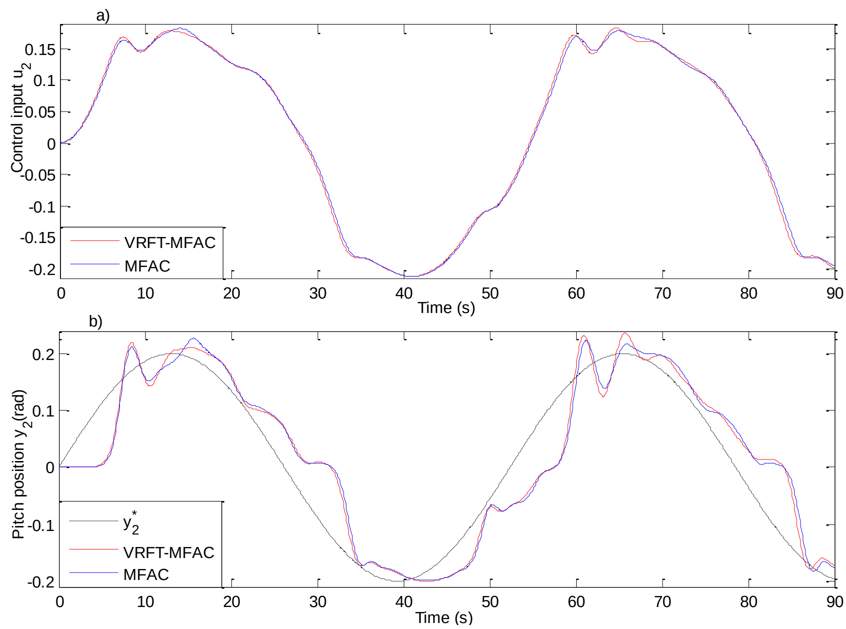

Figure 3
Experimental results related to SISO azimuth control: a) u_2 versus time, b) y_2 and y_2^* versus time

Conclusions

This paper has proposed the combination of two data-driven control approaches, which has led to the formulation of a mixed MFAC-VFRT control approach. This approach leads to mixed MFAC-VFRT algorithms that are actually MFAC algorithms tuned by VRFT.

The experimental results presented in Section 5 outline that the differences of the o.f.s. from Table 2 are insignificant for both azimuth and pitch SISO control. Figures 2 and 3 show that the control inputs and the controlled outputs almost overlap. Therefore, the mixed MFAC-VFRT control approach is a time saving solution that finds the controller optimal parameters and offers similar CS performance with that of CS with MFAC algorithm, whose initial parameters were obtained using GSA. The mixed MFAC-VFRT control approach is useful for processes whose identification is difficult or impossible.

Further research will treat the study of several constraints concerning the choice of the reference model and performance improvement, which can be achieved by the combination of artificial intelligence techniques (including fuzzy control) and neural networks [32]-[39].

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