

Non-Linear Behavior of Sands under Longitudinal Resonance Testing

**Merouane Mekkakia Maaza, Ahmed Arab, Mostefa Belkhatir,
Saaed Hammoudi**

Laboratory of Materials Sciences & Environment, University of Chlef (Algeria)
e-mail: mek_mer@yahoo.fr, Ah_arab@yahoo.fr, abelkhatir@yahoo.com,
hamoudisaad@yahoo.fr

Minh Phong Luong

Laboratory Solid Mechanics, CNRS, Polytechnic School of Palaiseau (France)
e-mail: luong@lms.polytechnique.fr

Abdelatif Benaissa

Civil Engineering Dept, University of Science and Technology of Oran (Algeria)
e-mail: dzbenaissa@yahoo.fr

Abstract: One of the fundamental features needed to evaluate soil response during earthquakes, is the study of controllable external variables that may affect the instability phenomena of granular materials under vibration, such as acceleration, frequency, the interaction of grains and their arrangements. Despite previous researches in this field, an understanding of these phenomena is still incomplete. A more accurate description of one of the phenomena that we will see, is how the resonance curve changes and how the jump occurs with the frequency change. For this purpose, a series of longitudinal resonance excitation laboratory tests were carried out on dry sandy soils with different grain size distributions (spread and tight) and different densities to identify the instability zone. This type of test may be assimilated to a system subjected to a forced excitation with damping. The test results confirm the existence of a non-linearity zone represented by a "jump" just after the resonance for tight-grained sand. Moreover, this study shows that the grains interact with the contact forces. Indeed, a slight local density increase induces more collisions and friction, and therefore more dissipation, creating a pressure drop that attracts the neighboring particles and finally a low damping.

Keywords: sand; resonance; dynamic; vibration; non-linearity; frequency; velocity

1 Introduction

The topic we develop in this laboratory investigation concerns the problem of earthquake hazards. Indeed, the earthquake appears on the ground surface in terms of soil vibrations, which may induce phenomena whose consequences can be devastating for both human and socio-economic field. The soil is an assembly of grains and particles much more complex than the regular assembly of the spheres used in the linear elastic theory of Hertz. However, the dynamic study of such soil assembly provides us with very basic information on these phenomena. In addition, direct contact between the grains plays an important role when the soil deposit starts moving.

Granular materials have been the subject over decades of a significant number of previous research works. According to Jae (1996) and Mue (1998), granular assemblies are random arrangements of rubbing grains with a geometrical disorder. Granular soil deposits are characterized by a non-linear response, making their overall behavior surprising and complex (Evesq 2002). The importance of this non-linear soil behavior is commonly accepted in the earthquake engineering field (Lopez Caballero 2003). And according to (Roscoe and Burland 1968, Hicher 1985 and 1996, Biarez and Hicher 1994, Maalej et al 2007), this has already been demonstrated earlier by different studies; the mechanical behaviour of sands in the range of small strains ($\varepsilon \approx 10^{-5}$) revealed a non-linear elastic behaviour, which depends on the evolution of the modulus of elasticity. Non- linear behaviour can be characterized by rigidity and the degree of non-linearity (Atkinson 2000). Miksic (2008) showed that the complexity of granular soil is strongly linked to the inherent disorder of these deposits, due to the heterogeneity of the contact forces between the grains.

For this purpose, we propose to study the nonlinear phenomenon whose effects are often considered disturbing, leading to spectacular effects. This analysis leads logically to the examination and description of the dynamic properties of the material in terms of transfer curves. For this, many researchers have undertaken studies on the linear and non-linear response of a vibration on solids. Dublin (1959) found that the shape of the curve acceleration response-frequency (resonance curve) depends on the amplitude and shape of excitement. Harris and Crede (1961) studied the phenomenon of non-linear resonance curves. It was shown that two types of jumps appear: one on the right of the peak and the other to the left of the peak. These jumps represent the region of instability of the system, and the position of the jumps depends on the direction of frequency (decreasing or increasing the frequency).

The studies of Anand (1966) on non-linearity show areas of instability on the resonance curve, and show that the excitation forces play an important role in the presence or absence of jumps. The instability zone is primarily due to the frictional forces, which results in a jump which can behave like a linear system by introducing dissipative elements (an increase in the damping) (Mathey R and

Rocard Y 1963). Valette and Cuesta (1993) carried out a vibration study on a vibrating cord; they found that the resonance frequency increases with the excitation force and the non-linearity, but decreases when the damping coefficient increases. According to Lalanne (1999), the jump of the resonance curve is unstable and therefore cannot represent the transfer function of a physical system. The sources of non-linearity particularly are: large displacements, dry friction and the non-linearity of the material (Girard and Roy 2003).

Despite all of these works, the problem of non-linearity of materials is still poorly understood; fundamental aspects remain to be solved, including not only the underlying causes of the non-linear phenomenon, but also the effects that occur. The tests with the sinusoidal forced vibrations with damping make it possible to clarify certain points, in particular the curve shape, the amplitude versus the frequency, and resonance frequencies, as well as the instability phenomena that take place when the excitation frequency is varied (Perez J. P 1995). It is therefore necessary to supplement existing research with specific investigations to identify clearly the phenomenon. For this purpose, we carried out an experimental program on two sands with different grain size distributions and with different initial densities to highlight another fundamental aspect of the non-linear behaviour of the soil. This aspect is studied through the influence of some factors affecting the shape of the resonance curve and the resonance frequency. We subject the sand specimen to vibrations with an acceleration range of excitation ($0.25g \leq \Gamma \leq 1g$) and a variable frequency sweeping velocity ($V = 0.2$ Hz/s and $V = 0.1$ Hz/s). We consider the existence of a confining pressure within the sample created by the vacuum (σ_c) equal to 100 kPa, using an electromagnetic vibrator called "vibrant pot".

2 Tested Materials

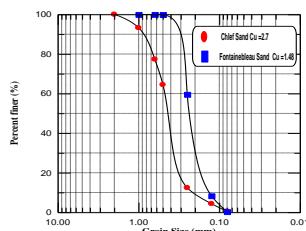


Figure 1
Grain size distribution curves of tested materials

The tested materials are granular materials. We used two different sands (Chlef and Fontainebleau). Figure 1 shows the grain size distributions of those two sands and Figure 2 illustrates the images taken by the scanning electronic microscope

(SEM), identifying clearly the texture of the studied sands. Table 1 presents their physical properties.

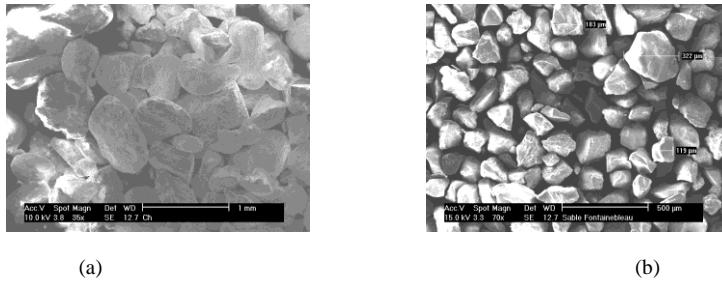


Figure 2
Image SEM: (a) Chlef sand and , (b) Fontainebleau sand

Table 1
Physical properties of tested sands

Material	γ_s (g/cm ³)	e_{max}	e_{min}	Grain shape
Chlef Sand	2.68	0.85	0.53	Rounded
Fontainebleau Sand	2.63	0.94	0.54	Mostly angular

3 Experiment and Materials Studied

3.1 Equipment

The apparatus used is an electromagnetic vibrator (vibrant pot) controlled by an electronic control unit of power. The capability of the vibrator (TW DERRITRON 3000) is 5 kN dynamic, with a range of sinusoidal frequencies between 20 and 10 kHz and a rack steering frequency, acceleration or speed imposed. A computer records and processes the data with suitable software developed at the Laboratory of Mechanics of Solids (Figure 3). Four sensors are connected to amplifiers and signal conditioners: one to measure the acceleration at the top of the sample (attached to the mass), a second for the acceleration at the bottom of the sample, a third to measure the force that is applied to the sample (under the brass plate), and a fourth connected to the pot to control the imposed acceleration. The mass is placed on the top of the sample that weighs 2850 g (Figure 4). The hammer is used to compact the samples to vary the density.



Figure 3
The electromagnetic vibrator connected to the electronic control unit

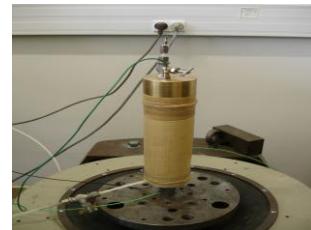


Figure 4
The Specimen placed on the vibrant pot

3.2 Preparation of the Samples and Procedure

The samples of dry sand that we used for a series of tests were prepared in a metal mold of 70 mm in diameter and 160 mm in height, on which was placed a rubber membrane. Pressed against the inner surface of the mold was a vacuum. We applied an air depression created by a vacuum pump (-100 kPa) through the opening, then we poured the sand into the mold in five layers of 200 g, and each layer was compacted. The various densities of the samples varied according to the number of compaction blows. Once the sample was placed at the initial density, a mass was placed at the top of the sample, applying a vacuum within the sample to allow its manipulation and to put it in an upright position (in equilibrium) on the pot. This was a very important point, because the pressure inside the sample was considered as a given pressure confined by compressed air σ_c (Figure 5). After reaching the desired vacuum, the tap was closed, and the mold and supports were removed. Finally, we had our sample perpendicular and in equilibrium (see Figure 4).

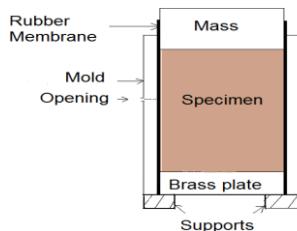


Figure 5
Sample preparation mold

3.3 Testing Procedure under Longitudinal Resonant Excitation

This test has been used by many authors including Hardin and Richart (1963) and Saada and Al (1978) together with the torsional resonance instrument to measure the velocity of longitudinal waves at the resonance of a cylindrical sample. Boelle (1983) developed this test to measure the Young modulus and Poisson's ratio at small strains ($\varepsilon \approx 10^{-5}$) (El Hosri 1984).

Our soil specimen to be tested was encased by a rubber membrane. It was then placed on a base attached to the oscillating diaphragm by a brass plate. A mass was placed on the top of the specimen, which was then placed under a vacuum, considered as a confining pressure. At the beginning of the test we applied a vibration with a frequency scanning varying from 300 Hz to 30 Hz (in decreasing the frequency), while the velocity and acceleration were imposed (see Figure 5). We could assimilate our sample to an oscillatory system by a single mass supported by a spring and damper (viscoelastic model). The support received an excitation, which is defined by an acceleration known (Γ); the excitement spread towards the mass through the elements K and C. The vibration that supports the mass translates into a response movement (Figure 6).

The program "pot" recorded time, scanning, acceleration at the top, acceleration at the bottom, and the dynamic force. It also displayed the amplitude and the signal phase of the acceleration at the top, taking into account the acceleration at the bottom, and the transfer curve.

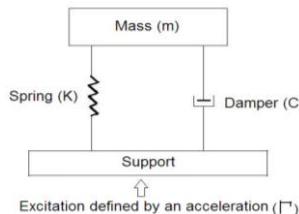


Figure 6
Schematic of a system-mass-spring-damper

4 Analysis of Experimental Results

4.1 The Influence of Particle Size on the Resonance Curve

Many authors have studied the influence of grain shape, grain size and mineralogy of the materials on mechanical properties at the resonant longitudinal column in the range of small deformations (Constantino 1988). Skoglund, Macurson and Cunny (1976) have studied the influence of soil structure on the modulus of the resonant column using two types of soil, sand and silt clay (Homsi, 1986). According to Luong (1986), we can see that by its morphology, a sandy soil acts as a filter frequency Low-pass.

The experimental program conducted in the laboratory involved a series of tests of longitudinal resonance on the material described above for various conditions of initial density and acceleration for a range of excitation ($0.25 \leq \Gamma \leq 1g$), with a speed and a confining pressure imposed. This allowed us to investigate on the shape and evolution of the resonance curve, and how the jump occurs according to the excitation for a given size. Figures 7 and 8 illustrate the qualitative aspect of the jump phenomenon that appears after the resonance in the form of a straight line, which explains: for a frequency value there is an infinity of responses.

Figure 7 shows the evolution of acceleration versus frequency. We note that different curves show a linear shape around the resonance for the sand well graded (diversified grain sizes) and that the frequencies of its resonance change gradually as the acceleration of excitation increases. We can then say that the Chlef sand ($C_u > 2$) can be characterized by its linearity during the resonance; we can say that it is easy to identify the dynamic parameters at each point of response curves. Figure 8 shows curves with a nonlinear form around the resonance represented by a jump, which varies according to the acceleration of excitement. We deduce that the uniformly-graded, tight-grained sand subjected to vibratory loading presents a nonlinear resonance curve resulting in a jump, and the calculations are very difficult to perform, especially for bandwidth.

However, the analysis of the behavior of granular soils at resonance (Figures 7 and 8) shows that the sand grains is a very complex assembly, and provides information that can be summarized in the following:

This linear behaviour for the graded sand can be explained by the fact that the agitated grains of sand settle down and form regular geometrical figures, although they are highly agitated under the effect of vibrations with maximum amplitude (resonance). The rearrangement of grains (slips and rotation) is fast, since the various sizes of grains and their round shape (see paragraph 2) facilitates the movement of these and come together during the vibration motion corresponding to low amplitude. The test results show that the graded sand follows a linear variation before and after resonance. This is shown in Figure 7.

However, there is a non-linearity around the resonance for the uniform graded sand ($C_u < 2$). It is more significant when the acceleration of the excitation is important ($\Gamma > 1g$). The results of the tests carried out on the Fontainebleau sand show unusual shapes of the resonance curves with the appearance of a decreasing jump just after the resonance (Figure 8) This phenomenon is explained by the variation of the acceleration inside the sample (large amplitudes of Γ) during the resonance, in addition to the direct contact between the particles, which plays an important role when the soil specimen is put in motion. When the amplitude becomes maximum (resonance) and the time is very short, the rearrangement of the grains occurs with difficulty due to their size (uniformity) and shape (angularity). The jump that appears represents the nonlinear behavior, and for a single frequency value corresponds to several values of acceleration (undefined number).

Thus, we can say that the factors leading to the non-linearity are: grain size distribution, their arrangements, and dry friction around the resonance.

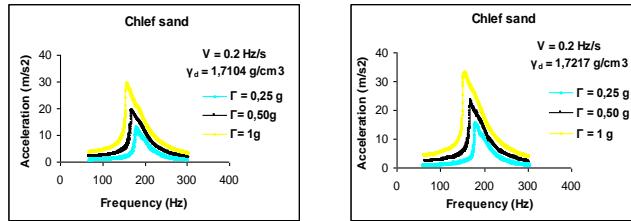


Figure 7
Resonance curves ($\sigma_c = 100$ kPa)

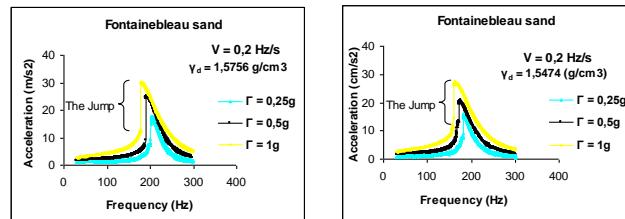


Figure 8
Resonance curves with an instability zone ($\sigma_c = 100$ kPa)

4.2 Effect of the Velocity on the Nonlinearity

The appropriate choice of scanning mode requires a scan rate slow enough so that the response reaches a large percentage of the steady-state response. Indeed, an extremely slow scanning allows for measuring and plotting the transfer function of

the system with a degree of freedom without distortion and for obtaining values of the resonance frequency and voltage (Lalanne, 1999).

To show the effect of velocity on the nonlinearity, we carried out tests on longitudinal resonance excitation on two different dry sands, Chlef sand (well graded) and Fontainebleau sand (poorly graded), under different densities and excitation speeds equal to 0.1 Hz/s and 0.2 Hz/s.

Curves 9 and 10 show that the sand of Fontainebleau presents an instability zone despite a decrease in the velocity from 0.2 to 0.1 Hz/s, but the jump varies in size according to the velocity: as agitation decreases, the jump decreases. Meanwhile, the Chlef sand maintains its linearity; then we can say that in this case the nonlinearity is independent of the velocity.

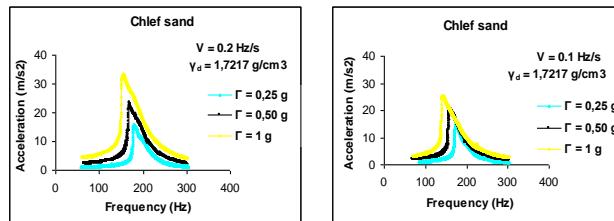


Figure 9

Resonance curves ($\sigma_c = 100$ kPa)

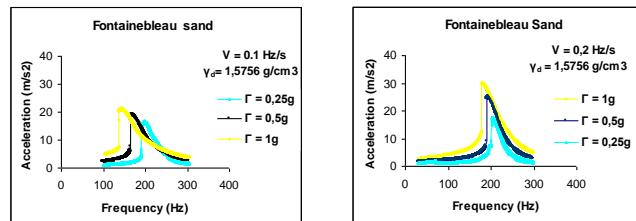


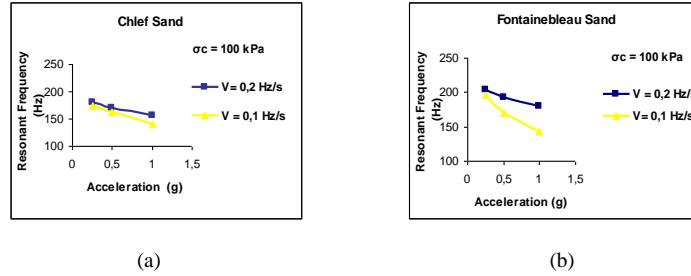
Figure 10

Resonance curves ($\sigma_c = 100$ kPa)

4.3 Effect of the Velocity on the Frequency of the Resonance

Our results (Figures 9 and 10) show that when the scanning speed is slow, the response curves of the acceleration-frequency (resonance curves) show a reduction in the maximum peak acceleration, a shift of the abscissa of the maximum, and a displacement of the center line of the curve (which loses its symmetry); as we remark that an increase of the bandwidth.

Finally, we can say according to the Figure 11 that the resonance frequency increases with the scan rate for all types of sand, and that the acceleration measured at the top of the sample depends on the scan velocity rate.



(a) (b)
Figure 11
Effect of propagation velocity on the resonant frequency

Conclusions

Our study has highlighted the nonlinear behavior of uniform sand ($C_u < 2$) under vibration loading. This nonlinearity appears around the resonance, which means that this step in the rearrangement of the grains is difficult because of the sudden agitation, uniform particle size and the angular shape of the grains. This is in contrast to the well graded ($C_u > 2$) sand, where the diversity and rounded shape of the grains facilitate their quick rearrangement during the resonance phenomenon, which guaranteed the non-appearance of the instability zone (jump). This type of nonlinearity appears on the resonance curves in a particular manner that is represented by a jump (a zone of instability). Our work has also led us to define the state of resonance frequency linear variation depending on the speed excitement and the resonance curve asymmetry. However, the variation of the scanning velocity has no effect on the nonlinearity of the material.

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Notation

$g (\text{m/s}^2)$: Gravitation

$\gamma_s (\text{g/cm}^3)$: Solid unit weight

$\gamma_d (\text{g/cm}^3)$: Dry unit weight

e_{\max} : Maximum void ratio

e_{\min} : Minimum void ratio

C_u : uniformity coefficient (Hazen coefficient)

C: Damper coefficient

K: spring constant

Γ : Acceleration

V: Velocity

σ_c : Confining pressure

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