

Multi-Objective Evolutionary Optimization Technique Applied to Propeller Design

Mojtaba Kamarlouei¹, Hassan Ghassemi¹, Koorosh Aslansefat², Daniel Nematy¹

¹Department of Ocean Engineering, Amirkabir University of Technology, Hafez Ave., Tehran, Iran, POBox: 15875-4413

²Shahid Abbaspur College of Engineering, Shahid Beheshti University, East Vafadar Blvd., Tehranpars, Tehran, Iran, POBox: 16765-1719

E-mails: m.kamarloie@aut.ac.ir; gasemi@aut.ac.ir; k_aslansefat@sbu.ac.ir; dani1760@aut.ac.ir

Abstract: Multi-objective functions of the propeller blade optimization are always regarded as important aspects of propeller design. This paper particularly presents a computational method to estimate the hydrodynamic performances including minimum cavitation, highest efficiency, and acceptable blade strength. The included parameters are as well, the number of blades, chord length, thickness, camber, pitch, diameter and skew. We also discuss the effect of the skew on the propeller performance and extract a formulation for these propose. In the optimization process, the evolution strategy (ES) technique is linked to the computational method to obtain an optimum blade. In order to allow the large variation of blade form during optimization process, the propeller section is represented by NURBS. New propeller forms are also obtained from the well-known B-series and DTRC are taken as initial forms in the optimization process at design speed of typical ships. The benchmark results for the two test cases prove the designed optimum propeller to be acceptable.

Keywords: Propeller performance; Optimization; Blade Design; Evolution Strategies

1 Introduction

Using the theoretical propeller design methods such as lifting-line or blade element theories, as well as a computer which ignores the geometry constraints seen in series propellers, naval architects always design an optimized propeller. However, series propellers are still valuable and widely used in the early design of light or moderate loading propellers. Moreover, for anyone who cannot supply lifting-surface software, the traditional series propellers could be a good choice. There exists a huge series of propeller design among the propeller series, the most

common of which is the B-series. The other series including the Gawn series, Japanese series, KCA series, Lindgren series, Newton-Rader series, Wageningen nozzle series and many others are more or less used [1].

Propellers theories have significantly improved during the last decades and today several methods are available for propeller design and for analysis based on different levels of complexity. Before the computational era, the momentum theory of propeller or so called “actuator disk theory” which was the first analysis method, introduced by Rankine, Greenhill and Froude was common. Later the propeller blade element theory was proposed by Froude, Taylor and many others. Nowadays, the computational fluid dynamic (CFD) has become a common way in the design process due to its lower model production costs. Lifting line theory, lifting surface, panel methods, and RANS are some important numerical approaches for analyzing the propellers. At the top of these methods, the three-dimensional viscous flow models can be found, where the three-dimensional incompressible Reynolds-averaged Navier-Stokes (RANS) equations are implemented and solved iteratively. The lifting surface methods in advance incorporate RANS equations to account for the viscous effects near the blade walls. Grid generation technology has developed to discretize complex geometry. Results from these methods have a good agreement with experimental results for the open water characteristics [2].

In this paper, a computer code has been developed using MATLAB software, in which the propeller basic coefficients are calculated by blade element theory. Propeller geometry and its geometrical properties including area of each section, volume, mass and center of gravity for each blade have also been calculated. These parameters are then used for calculating the stress in blade sections, creating the geometry of the optimum propeller and finding the optimum characteristics of the B-series, while considering constraints is indicated in this paper. The propeller design process is treated as a multi-objective function subjected to several constraints including minimum cavitation, highest efficiency and highest thrust, however higher skew, lowest torque, and an acceptable blade strength are also guaranteed.

Literatures on ship propeller optimization research are in fact extensive. First, an investigation on the possibility of maximizing the efficiency by utilizing Genetic Algorithm (GA) was done by Lee and Lin [3]. Later on, Plucinski *et al.* optimized a self-twisting propeller, using a Genetic Algorithm by considering the orientation angles of the fibers in each layer as the design variables of efficiency improvement for an optimum design [4]. A propeller performance analysis program was also developed and integrated into a genetic algorithm by Christoph Burger [5]. Matulja and Dejhalla found optimum propeller geometry by using artificial neural network [6]. Chen and Shih designed an optimum propeller by considering the vibration and efficiency in B-series using Genetic Algorithm [7]. Emmerich *et al.* worked on Design Optimization of ship propellers using

Metamodel-assisted evolution strategies. They compared different methods to find how to accelerate evolution strategies by means of metamodels on artificial test problems similar to the time consuming evaluation function [8]. Xie proposed a multi-objective optimization approach for propeller preliminary design. The objectives were both efficiency and thrust coefficient [9]. Koronowicz et al. released a computer program which was capable of conducting complete design calculations of ship propellers, including their analysis in a real inflow velocity field behind the ship hull [10]. Likewise, Cho and Lee developed a numerical optimization technique to determine the optimum propeller blade shape for efficiency improvement. Their method faces the constraints of the constant power coefficient and work condition [11]. In addition, Vesting and Bensow worked on an optimization of a propeller blade with the propeller operating in behind conditions while considering sheet cavitation. They also took in to account the effect of the propeller on the flow field around the stern of the ship [12].

In theoretical view, both marine and aircraft propeller work in the same way, but a marine propeller operates in a much dense fluid compared to that of aircraft that operates in air, so it experiences more stress compared to aircraft propellers which makes it more difficult to move through water. Also, a marine propeller can experience cavitation which, in severe condition, can lead to erosion and performance decay as a result of thrust break down [13]. The techniques of propellers strength calculations have not changed in essence, since the developments of the propellers in early 1970s. The first method was the cantilever beam theory which is still being used these days as cornerstone of the propellers calculations. This method was developed by Admiral Taylor in early 20th Century and since then it has been developed, and is the prominent method used in this paper.

The main purpose of this paper is to design a propeller using the blade element theory that could generate the desired thrust with the lowest torque, highest efficiency and an acceptable blade strength with no cavitation. Here, we also discuss the influence of the skew on the propeller characteristics such as efficiency, cavitation and strength. A US research has been done on the influence of the skew on cavitation and propellers characteristics in the naval ship research and development center in 1971 by Robert J. Boswell [14]. In this research the effect of the skewed propellers on the speed at which cavitation begins, and propeller performance in both forward and backward conditions has been investigated. It is clear that the high skew of propeller may reduce the cavitation, thus this is the one big advantage with skew. Yet, finding a comprehensive formula for this is left to future investigations. Therefore, a computer program is designed to generate the blade geometry, calculate the propeller performance, and measure stress in the blade section. Then, a genetic algorithm is used to achieve the best trade-off between indicated objectives.

2 Methodology

The basic theories used in this paper are the Blade Element Theory (BET) for the blade characteristic, the cantilever beam theory for calculating the blade strength, Keller cavitation method and Bucket diagram for cavitation analysis, and finally, Multi-objective Genetic Algorithm for the optimization process.

2.1 Blade Element Theory

The BET is interested in how a propeller generates its thrust and how this thrust depends on the shape of the propeller blades. A propeller is assumed to be a combination of a series of blade elements, each of which produce a hydrodynamic force due to their motion in the fluid. The axial component of this hydrodynamic force is called “thrust” while the moment about propeller axis of the tangential component is called “torque”. Integrating the thrust and torque components over the radius of the propeller for all blades gives total thrust and torque for the propeller.

If a blade is divided into a large number of elements, each of these elements is then treated like foil subjected to an incident velocity V_R as shown in Figure 1. The resultant velocity was considered to include an axial velocity V_A together with a rotational velocity ωr , which clearly varies up to the blade tip. In normal working conditions, advanced angle β_i is less than the blade pitch angle ϕ at the section, hence the section has an angle of attack α . Thus, because of the combination between the zero lift angle of the foil and angle of attack the section will experience lift and drag forces. For a given section, the elemental thrust and torque are measured by;

$$dT = \frac{1}{2} \rho z C V_A^2 (c_l \cos \beta - c_d \sin \beta) dr \quad (1)$$

$$dQ = \frac{1}{2} \rho z C V_A^2 (c_l \sin \beta - c_d \cos \beta) r dr \quad (2)$$

Where, z is blade numbers and C is the chord length.

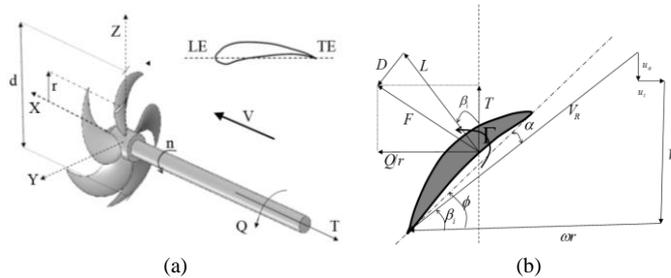


Figure 1

Coordinate system of propeller (a) and Inflow velocity and hydrodynamic forces acting on the blade at radius r (b) [15]

Now the efficiency of the section η is measured by

$$\eta = \frac{VdT}{\Omega dQ} \quad (3)$$

Consequently, this propeller-theoretical model allows the thrust and torque to be calculated, provided that the appropriate values of the lift and drag are known [15]. The result of BET compared with experimental data for four-blade propeller [16] are illustrated in Figure 2 (a, b and c).

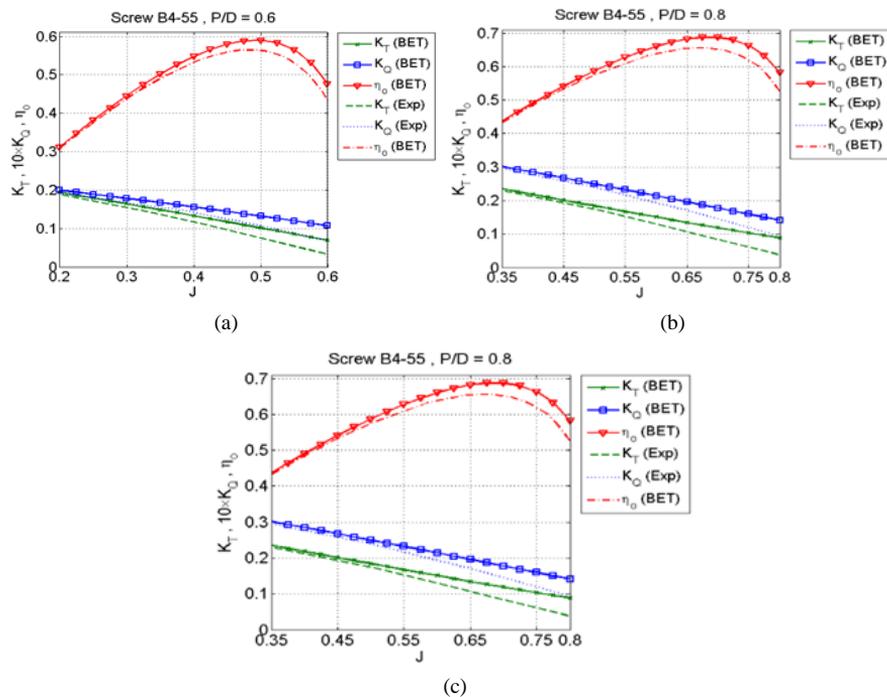


Figure 2

Comparison between experimental and predicted performance of Wageningen B-screw series propellers. Pitch ratio is shown for (a), (b), and (c) as 0.6, 0.8, and 1.2 respectively.

3 Optimum Design

In order to design an optimum propeller, some constraints could be considered as objective functions which are used in multi-objective genetic algorithm. The constraints used in this paper for design and optimization of the propeller are mentioned below.

3.1 Cavitation Constraint

One of the most widely used cavitation criteria for marine propellers is a diagram first introduced by Burrill (1943). This diagram gives the limit value of a thrust loading coefficient τ_C as a function of the cavitation number $\sigma_{0.7R}$. Another criterion which may be used to determine the expanded blade area required to avoid cavitation is based on Keller's (1966) [17]. It is generally known that cavitation could affect a propeller's performance and need to be considered during the design process. A simple way to mitigate cavitation is to increase the blade area ratio. Here, the Keller criteria is employed as follows:

$$\left[\frac{A_E}{A_O} \right]_{min} = \frac{(1.3 + 0.3z)T}{(P_O - P_V)D^2} + K \quad (4)$$

where, $(A_E / A_O)_{min}$ is the minimum blade area ratio, the coefficient K equals 0.1 for twin propeller, and 0.2 for single propeller.

Although cavitation-free propellers have been successfully designed for decades using simple cavitation criteria such as those of Burrill and Keller, it must be realized that cavitation depends not only on the thrust loading and the cavitation number, but also on the non-uniformity of wake and the detailed geometry of the propeller blade sections. Cavitation characteristics of airfoil sections have therefore been determined as a function of the thickness-chord ratio and the angle of attack for different camber ratios and thickness distributions. The diagram which satisfies this method was named as Bucket diagram.

Therefore, both Keller and Bucket criteria are considered for cavitation analysis in this paper. shows the Bucket diagram for two optimized propeller.

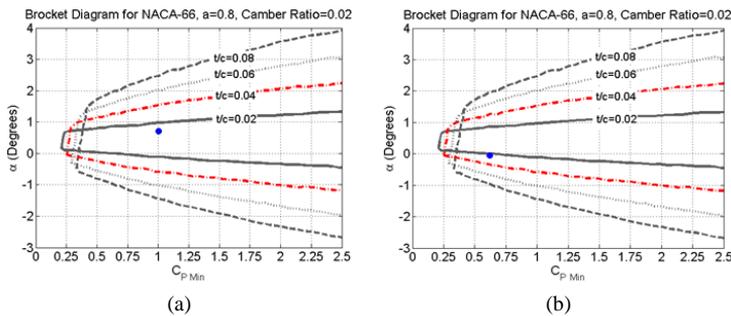


Figure 3

Bucket diagram [18]; (a) Bucket diagram for OP-101, no cavitation accrues in $r/R = 0.7$, (b) Bucket diagram for OP-102, no cavitation accrues in $r/R = 0.7$

3.2 Strength Constraint

The forces acting on a propeller are generated from the thrust and torque of the propeller and the centrifugal force on each blade caused by its revolution around the axis. Due to the complex shape of the propeller blades, the accurate calculation of the stress resulting from these forces is extremely difficult. The effect of the ship maneuvering on forces acting on the propeller as well as the effect of the ship oscillating speed on propeller loading are also difficult to estimate. Even in the calm water condition, due to the effects of the varying wake, the loading on a propeller in every revolution varies. This loading condition and the effect of the residual stress that may remain during the manufacture of the propeller as well as the effect of the corrosion and erosion would make the estimation of the propeller stress much more difficult. In practice, therefore, it is usual to adopt fairly simple procedures based on a number of assumptions to make the problem less complex, and also to make sure that the stress calculated by these simplifications is in a quite good agreement with the experiment results. Some of these assumptions are as follows: 1) Each blade is assumed to be like a beam cantilever to the boss. 2) The stress distribution along the chord is ignored and is only considered along the radius. 3) The calculations are considered according to the ship constant speed. 4) The bending moment acting on a blade are also assumed to act on a cylindrical section. 5) The stress at the section is calculated on the basis of the simple theory of the beam, the neutral axis is parallel and perpendicular to the chord of the expanded section.

Admittedly, due to thrust and torque on the blade, the bending moments are [17]:

$$M_T = \int_{r_0}^R \frac{1}{z} \frac{dT}{dr} (r - r_0) dr \quad (5)$$

$$M_Q = \int_{r_0}^r \frac{1}{rz} \frac{dQ}{dr} (r - r_0) dr \quad (6)$$

where, dT and dQ are the thrust and torque of an element between r and $r+dr$. Also the consequent bending moment due to centrifugal force is [17]:

$$F_C = m_b \bar{r} (2\pi n)^2 \quad (7)$$

where, $m_b = \int_{r_0}^R \rho_m a dr$ is the blade mass, and $\bar{r} = \frac{\int_{r_0}^R ar dr}{\int_{r_0}^R a dr}$ is the centroid. So the

moments due to centrifugal force are:

$$M_R = F_C \cdot Z_C \quad (8)$$

$$M_S = F_C Y_c \quad (9)$$

where Y_c and Z_C are the space between the centroid of the blade with centroid of the section. M_R and M_S are the moments due to rake and skew angels, respectively. So the stress in section is:

$$S = \frac{M_{x0}}{I_{x0}/y_0} - \frac{M_{y0}}{I_{y0}/x_0} + \frac{F_C}{a_0} \quad (10)$$

where $M_{x0} = -(M_T + M_R)\cos\varphi - (M_Q + M_S)\sin\varphi$ and $M_{y0} = (M_T + M_R)\sin\varphi - (M_Q + M_S)\cos\varphi$ which, I_{x0} and I_{y0} are the section muduluses about the x_0 and y_0 (axes of the centroid of the section) and a_0 is the area of the section. It is obvious that the cantilever beam theory is a simple method to estimate the maximum tensile or comparison stress in any blade section. For doing the above-mentioned procedure we first of all create a propeller geometry and then divide the blade sections into 26 stations in chord direction and 11 sections in radial, thereafter we do integrating by Simpson methods for calculation of the volume, momentum of inertia and area for the procedure, then calculate the moments of thrust and torque and at the last step estimate the stress in blade sections (root, 0.25R and 0.3R). The amount of stress achieved by this method should be less than maximum allowable stress of the propeller material. It is noted that, the propeller material in this paper is considered as nickle mangeneze bronze allay [18].

In order to achieve a proper blade thickness and to ensure the blade strength, the following formulation can be used to determine the minimum thickness ratio at 0.7R [18]:

$$\left[\frac{t_{min}}{D} \right]_{0.7R} = 0.0028 + 0.21 \sqrt{\frac{[3183.87 - 1508.15(P/D)]P_S}{1266652.04nD^3(S_C + 20.9D^2n^2)}} \quad (11)$$

where, $\left[\frac{t_{min}}{D} \right]_{0.7R}$ is the blade minimum thickness, and S_C is maximum allowable stress of the propeller material in *MPa*. According to B-series propeller geometry [19], the blade's maximum thickness ratio at each section relative to the propeller diameter is given in Table 1.

Table 1
blade thickness % of D for B-series propellers [1]

r/R	Max. blade thickness (% of D)		
	z = 3	z = 4	z = 5
0.2	4.06	3.66	3.26
0.3	3.59	3.24	2.89
0.4	3.12	2.82	2.52
0.5	2.65	2.4	2.15
0.6	2.18	1.98	1.78
0.7	1.71	1.56	1.41
0.8	1.24	1.14	1.04
0.9	0.77	0.72	0.67
1	0.30	0.3	0.3

By using the equation 11 and the geometry of the B-series propeller the required blade thickness is obtained as follows:

$$\left[\frac{t}{D} \right]_{0.7R} \geq \left[\frac{t_{min}}{D} \right]_{0.7R} \quad (12)$$

3.3 Maximum Efficiency

The calculated propeller thrust (T_{Cal}) must be equal or more than total ship resistance. The propeller thrust and the minimum required thrust (T_R) can be calculated as follows:

$$T_{Cal} = K_T \rho n^2 D^4 \quad (13)$$

$$T_R = \frac{R_T}{n_p (1 - t_{de})} \quad (14)$$

where, R_T is the total ship resistance, n_p is the number of propeller and t_{de} is the thrust deduction factor. Then K_T is used in calculations as follows:

$$K_T = A J^2 \quad (15)$$

where, J is propeller advance ratio and A is indicated in Eq.(16)

$$A = \frac{T_R}{\rho \times V_A^2 \times D^2} \quad (16)$$

Then J is achieved through Eq. (15) and then K_Q and $\eta_o = \frac{K_T}{K_Q} \times \frac{J}{2\pi}$ are achieved through BET code and resulted curves.

3.4 Effect of Skew

An effective measure for diminishing cavitation, vibratory pressures and shaft forces is to employ extreme skew. It is obvious that when the blades are sufficiently skewed, the sections gradually pass through the crest of the wake thus causes the oscillating forces to reduce.

In addition to the above-mentioned advantages, the skew causes a decrease in the efficiency. The effect of the skew on the propeller efficiency indicates that an approximate formula may be obtained for efficiency in terms of the skew angle [20].

$$\frac{\eta_{Skew}}{\eta_o} = 0.06687e^{-0.1148\theta_s} + 0.989e^{-0.001029\theta_s} \quad (17)$$

where, θ_s is the skew angle in degrees and η_o is the openwater efficiency. Figure 4 shows the efficiency of the skewed propeller versus skew angle.

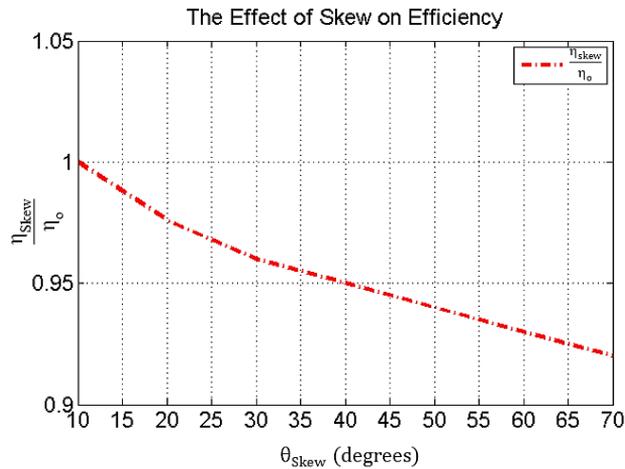


Figure 4
Effect of skew on the propeller efficiency based on Eq.(16)

3.5 Flowchart Calculation Method

Based on the above-mentioned descriptions of all constraint equations and MBET, the flowchart is presented in Figure 5. As it can be seen in this figure, the ships data are first given as inputs. Propeller optimum geometry may also be achieved through the iterative method in order to cover the constraints and objective functions.

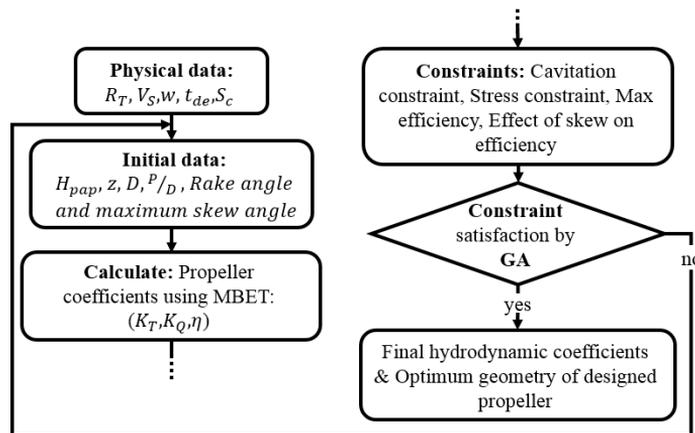


Figure 5

Optimum propeller calculation flowchart

4 Genetic Algorithms

The main difficulty in most optimization problems does not lie in mathematics or related methods, but mostly in formulation of the constrain objectives. The propeller optimization problem can be classified as a multi-objective constrained one. Evolutionary Algorithms are in fact non-classical methods that do not fall into the trap of local minimums. One of the most famous methods is named genetic algorithms, known as a method to find optimal solutions. In this method, the input variables (z , D , θ_{skew} , P/D) are assumed as genotype and output variables ($1/K_t$, K_q , EAR and $1/\eta_{skew}$) as phenotype on both of which the genetic operations are applied. In each generation, selection functions pick the most significant genes up as the parents of the next generation and then the crossing over procedure is performed on them. Among these, the random genes are added to the population as mutation functions and this procedure is repeated until ultimate criteria are established. Different conditions can be set to stop the problem. In this paper, the condition was to reach the number of iterations which

is set to maximum 550. The flowchart of the optimization process approach is shown in Figure 6 [21].

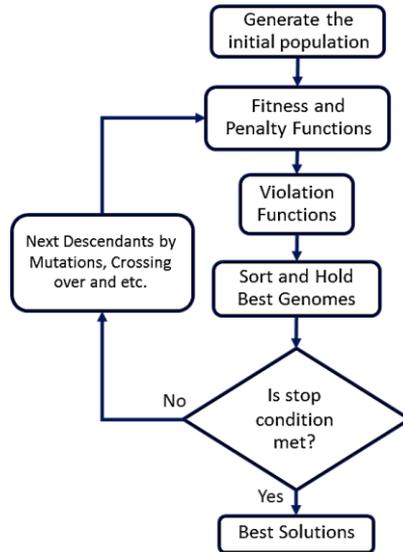


Figure 6

Flowchart of the process optimization approach [21].

Any evolutionary optimization algorithm needs to be configured by settings. Parameters for this paper are shown in Table 2.

Table 2
Setting of Genetic Algorithm

GA settings	
Type of parameter	Rate or type of consideration
Population Size	40
Iteration or Decades	550
Percentage of Mutations	35%
Type of Mutations	Random Number Generation
Percentage of Crossover	50%
Type of Crossover	2 Point Crossing Over
Percentage of Recombination	15%
Type of Selection	Random Selection

In the cost function all output variables are normalized and constraint of η_o conditions applied with penalty function by Eq. (18) as follows:

$$V \{ \eta_o \leq 0.6 \} = \begin{cases} 0 & \eta_o \leq 0.6 \\ \frac{\eta_o}{0.6} - 1 & \eta_o > 0.6 \end{cases} \quad (18)$$

Note that in this paper two type of constraint conditions are applied, the first type can be called input constraint which are addressed in Table 4 and the second type can be called output constraint which is addressed in Eq. (18).

5 Case Study

Table 3 shows two different conditions designed by the propeller. Furthermore, some limits can be established as inputs which are indicated in Table 4.

Table 3
Considered design condition

Ship code	ship speed (Knots)	wake factor	thrust deduction facture	total resistance (KN)
V-101	16	0.0506	0.0731	57.68
V-102	27	0.0506	0.0731	200.27

Table 4
Boundary constraints

Design variable	Lower limit	Upper limit
Number of blades	3	7
Skew angle (θ_s), Degrees	50	108
Maximum Allowable Stress (S_c), MPa	-	39 (Depend on material)
Pitch ratio	0.5	1.4
Propeller advance ratio	0	1.5

The final results are illustrated in Table 5 which includes eight variables. Meanwhile, the trend of each parameter during optimization process is shown in figures for both optimized propellers (OP-101 and OP-102). The most significant feature of these figures is the mutation occurred during the optimization. It should be indicated that P-101 and P-102 are two propellers designed for conditions mentioned in Table 3 in our previous work [22] with no optimization process and without considering skew effects and stress consideration. So, efficiency of P-101 and P-102 are not affected by skew impacts.

Table 5
Output results of BEM method [22] and present developed optimization program

Propeller code	Blade Number	Diameter (m)	P/D	θ_s (Degrees)	Thrust (kN)	Torque (kN.m)	Efficiency	Max Stress in Root (MPa)
P-101	4	2	1.2	15	59.400	24.150	0.608	-
P-102	4	2.2	1.18	12	212.650	122.330	0.600	-
OP-101	3	2.008	0.617	55.002	62.210	17.205	0.570	8.135
OP-102	5	2.900	0.787	55.884	216.068	116.403	0.565	7.242

The variation of the thrust can also be seen in. While shows the torque variation. The stress in root section can be monitored in. It is generally known that the amount of thrust, torque and skew angle as well as other blade design parameters would affect the stress in each section. illustrates the change in maximum efficiency. Also, the effect of skew angle on efficiency can be monitored in the same figure.

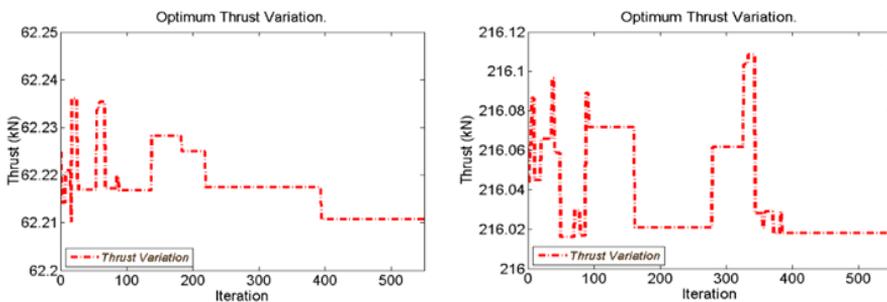


Figure 7
Thrust variation during optimization

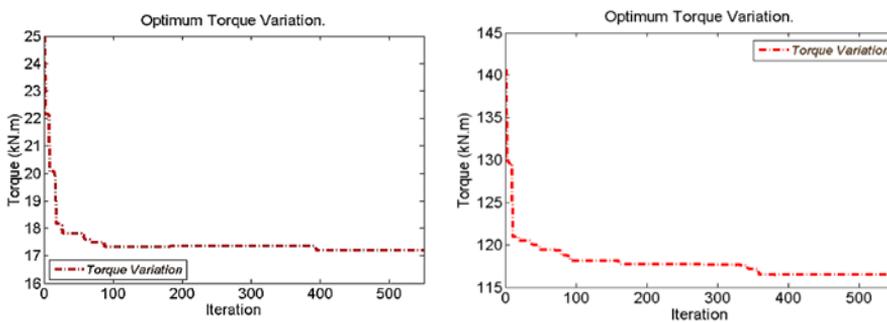


Figure 8
Torque variation during optimization

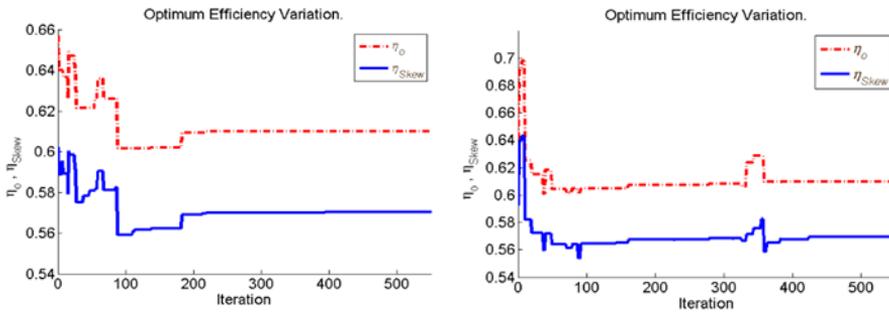


Figure 9

Efficiency variation during optimization, η_{Skew} shows the variation of efficiency effected by Skew variation

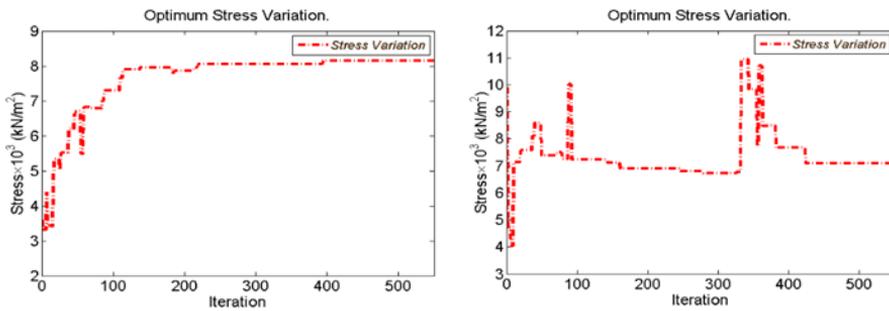


Figure 10

Stress variation during optimization

The geometry definitions of both optimum propellers are shown in Table 6 and Table 7, including the distribution of chord, thickness, camber, and skew along the blade radius. Finally, their 3D Geometry are plotted in Figure 11 and Figure 12.

Table 6

Geometry definition of P-101 propeller

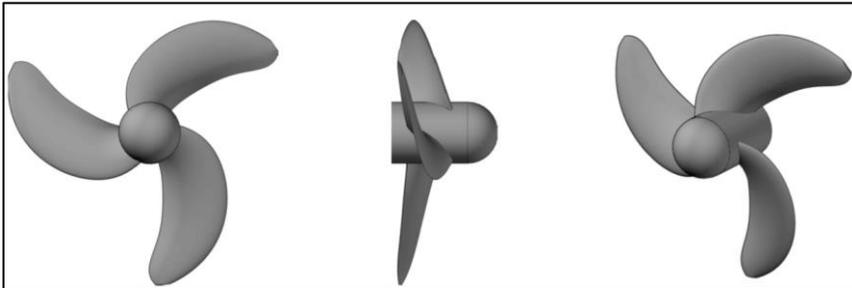
r/R	c/D	θ_s (Degrees)	t/c	f/c
0.2	0.208	0.000	0.175	0.040
0.25	0.220	3.549	0.156	0.041
0.3	0.233	7.099	0.139	0.041
0.4	0.257	14.371	0.110	0.039
0.5	0.269	21.382	0.088	0.034
0.6	0.271	28.088	0.071	0.028
0.7	0.266	34.721	0.058	0.023
0.8	0.248	41.437	0.047	0.019

0.9	0.194	48.202	0.036	0.016
0.95	0.137	51.586	0.031	0.017
1.0	0.000	55.000	0.025	0.000

Table 7

Geometry definition of P-102 propeller

r/R	c/D	θ_s (Degrees)	t/c	f/c
0.2	0.209	0.000	0.175	0.040
0.25	0.220	3.553	0.156	0.041
0.3	0.234	7.104	0.139	0.041
0.4	0.259	14.386	0.110	0.039
0.5	0.271	21.398	0.088	0.034
0.6	0.272	28.116	0.071	0.028
0.7	0.267	34.813	0.058	0.023
0.8	0.249	41.516	0.047	0.019
0.9	0.194	48.312	0.036	0.016
0.95	0.137	51.646	0.031	0.017
1.0	0.000	55.884	0.025	0.000

Figure 11
3D geometry of P-101

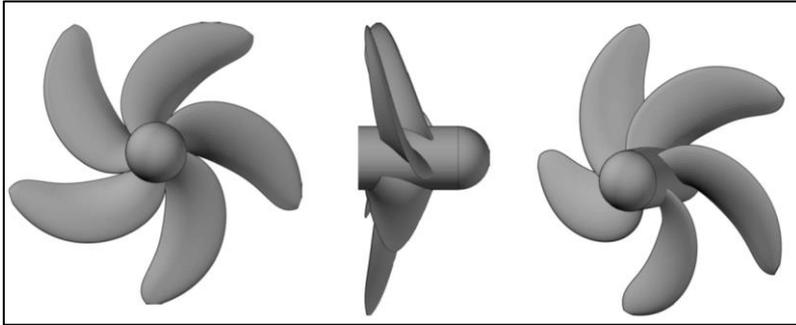


Figure 12
3D geometry of P-102

Conclusions

This paper presents the propeller design by using some important constraints and GA techniques based on numerical results. Therefore, the following conclusions can be drawn.

1. The present lifting line theory is relatively satisfactory for the propeller characteristics at various pitch ratios.
2. The present propeller design is considered based on 4-constraints technique simultaneously which proves the final designed propeller to be more reasonable and practical.
3. The skew effect is a new practical constraint to estimate the propeller efficiency for limiting the cavitation problem. This constraint is the most important one in the present computational method.
4. This research can be extended to the other meta-heuristic algorithm and then take a discussion and comparison about efficiency, fastness, robustness and etc. In additions, the other propeller parameters and their effort can be considered as variable of optimizations.

Acknowledgement

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Nomenclature

A_E	Propeller expanded area, m^2	Q	Torque Force, kN
A_O	Propeller disk area, m^2	R_T	Total resistance, kN
a_0	Root section area, m^2	\bar{r}	Root to center of mass of the blade, m

C	Chord, m	S_c	maximum allowable stress of the propeller material, MPa
c_l	Lift coefficient	T	Thrust force, kN
c_d	Drag coefficient	TE	Trailing edge
$C_{P\min}$	Minimum pressure coefficient	T_R	Required thrust, kN
C/D	Chord ratio	t_{de}	Thrust deduction factor
D	Propeller diameter, m	t/D	Thickness ratio
F_C	Centrifugal force, kN	u_a	Axial induced velocity (m/s)
H_{pap}	height of propeller aperture, m	u_t	Tangential induced velocity (m/s)
I_{x0}	Section modulus against x axis, m^4	V_s	Ship speed (V_R), (m/s)
I_{y0}	Section modulus against y axis, m^4	V_A	Advance speed, (m/s)
J	Advance ratio	w	Wake factor
K_t	Thrust coefficient	z	Number of propeller blades
K_q	Torque coefficient	β	Hydrodynamic pitch angle, degree
K_p	Chord factor	α	Angle of attack, degree
L	Lift force	ϕ	Geometrical pitch angle, degree
LE	Leading edge	Ω	Section rotational speed, rad/sec
M_T	Thrust moment, kN.m	θ_s	Skew angle, degree
M_Q	Torque moment, kN.m	η_o	Openwater Efficiency
M_R	Moment due to rake angle, kN.m	η_{skew}	Efficiency affected by Skew
M_S	Moment due to skew angle, kN.m	ρ	water density, kg/m^3
n	Propeller rotational speed, rps	ω	Propeller rotational speed, rad/sec
P/D	Pitch ratio		

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