# Geometric Error Correction in Coordinate Measurement

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*Abstract: Software compensation of geometric errors in coordinate measuring is hot subject because it results the decrease of manufacturing costs. The paper gives a summary of the results and achievements of earlier works on the subject. In order to improve these results a method is adapted to capture simultaneously the new coordinate frames in order use exact transformation values at discrete points of the measuring volume. The interpolation techniques used have the draw back that they could not maintain the orthogonality of the rotational part of the transformation matrices. The paper gives a technique based quaternions which avoid this problem and leads to better results.*

## 1 Introduction

Three dimensional coordinate metrology is a firmly established technique in industry. Their universal applicability and high degree of automation accounts for it's success in the last 30 years. In order to full-fill its task to verify the geometry of products on the basis of the measured results CMM-s must be in principally be an order of magnitude more accurate than the machine tool used to manufacture the part.

Over the last 50 years one can observe enormous enhancement in positioning and measuring accuracy. The main portion of this enhancement is the result of improved knowledge about high precision machine design [18]. A fundamental principle was recognized by professor Abbe already in the 1890's about the alignment of the displacement measuring system with the distance to be measured. Another fundamental principle is the separation of the structural and measuring functions in a machine. Already in the 1880's measuring equipment was built in witch the measuring system was attached to a separate metrology frame. The third important factor, to be concerned, is the thermal distortion of the metrology system. A short overview of novel constructions for high precision coordinate measuring machines is given in [8].

As mechanical accuracy is costly, whereas repeatability is not expensive, software techniques were used from the beginning to compensate for the systematic errors in order to keep manufacturing costs low.

One of the earliest papers on error compensation of coordinate measuring machines is by Zhang et al. [25]. They describe the compensation of a bridge type industrial three-coordinate measuring machine, which resulted in an accuracy improvement by approximately a factor 10. The machine consists of only translational axis and the infinitesimal rotation errors are described by the rotation matrix where the trigonometric functions are replaced by the first term in their Taylor series. The correction vectors are determined at equally spaced points in the measuring volume and are stored in the memory of the computer in the form of look-up table. The correction vectors at intermediate point are calculated simply by linear interpolation.

An analytical quadratic model for the geometric error of a machine tool was developed by Ferreire and Liu [6] using rigid body kinematics. They introduced the notion of shape and joint transform. The former describes the transformation between the coordinate system on the same link and latter the transformation across a joint. To represent the transformations they introduced the use of homogeneous transformation in matrix form. A quadratic expression was developed for the case where the individual joint errors vary linearly with the movement along the joint or axis. The global error description was obtained by concatenating these matrices.

Duffie and Yang [2] invented a method to generate the kinematic error functions from volumetric error measurements. To represent the displacement error a vectorial approach was followed. The rotational errors were described by matrices in which, taking into account that the angular error are small, the cosine and the sine terms can be approximated with the lowest order terms of their Taylor series. The model was used to describe the error of a measuring probe neglecting the rotation. The translational error components were approximated by cubic polynomials. To find the coefficients least square fit was applied.

Teeuwsen [20] described the error motion of the kinematic components of a coordinate measuring machine by using homogeneous transformations and concatenating these transformations to calculate the resulting global error. Assuming that the rotational errors are very small, he neglected the second order term. Hereby he could ensure the commutativity of the matrices, but at the same time the orthonormality of these matrices was lost, which means that they do not represent a pure rotation anymore. The error motions of the probe displacements were also handled by transformation matrices. In order to establish the error map in the form of correction vectors the various error components were measured on a semi automatic way at discrete points of the measuring volume. To obtain a continuous description of the correction vector, between these points, regression was used to establish a piecewise polynomial representation.

Ruijl [15] has built a high precision coordinate measuring machine with a measuring uncertainty of 50 nm in a 100x100x40 mm measuring volume. The machine has a novel construction, where the air bearing table performs the measuring motion in all the three principal directions. It was derived that if the measuring systems are aligned with the centre of the probe tip the relationship between the position of the measuring system and the contact point on the workpiece is unique. This means that the functional point is the centre of the probe tip and hence it is possible to comply with the Abbe principle.

The nano measuring machine of SIOS [17] is based on the same principles. This machine is currently applied as the stage for a long range scanning microscope.

Kim et al. [12] have constructed an unusual machine. One attempt to get rid of the parallax error of orthogonal type coordinate measuring machines is the application of the so-called multilateration. It is to measure the diagonal distances of the probe using tracking laser interferometers with retro-reflectors. The paper describes a scheme of multilateration based on a single volumetric interferometer system. The volumetric interferometer generates two spherical wavefronts from the probe by using diffraction point sources. The emanated wavefronts interfere within the measuring volume, while two dimensional array of photodetectors mounted on the machine frame capture the interferometric intensity field. Phase information is used, from which the coordinates of the probe are determined. A second interferometer is installed to measure the x and y position of the machine table.

Kim and Chung [11] also applied infinitesimal matrix transformation to correct the position error due to geometric imperfections and transient thermal errors of a machine tool to improve on machine measurement accuracy. Thermal errors were derived from the thermal drift of the spindle in the three principle directions.

The static and transient thermal errors and their compensation are discussed by Kruth et al. [13]. Capturing temperature distribution of the machine structure the thermal deformations can be calculated using the linear thermal expansion coefficients of the individual machine components. However the determination of sensor positions in a cumbersome ‘trial and error’ task.

Recently in a paper Tan and his coauthors [19] describe the application of neural networks for the error compensation of single-axis, a gentry and X-Y stage. The advantage of using neural networks is in the followings: they could be used to approximate any continuous mapping. This mapping can be achieved by learning. Parallel processing and nonlinear interpolation can also be performed. Using this technique the authors could improve the positioning accuracy depending on the configuration investigated by a factor between two and three.

# 2 Overview of the Errors and their Sources

When considering the mechanical accuracy of coordinate measuring devices three primary sources of quasi-static errors can be identified:

- Geometric errors due to the limited accuracy of the individual machine components such as guideways and measuring systems.
- Errors related to the final stiffness of those components, mainly by moving parts.
- Thermal errors as expansion and bending of guideways due to uniform temperature changes and temperature gradients.

Geometric errors are caused by out of straightness of the guideways, imperfect alignment of the axis and flatness errors.

Deformations in the metrology frame introduce measuring errors. During measurement the deformation of the metrology frame is caused by the probing force. Its effect can be predicted with relatively high precision if the probing force is known and therefore it easily incorporated into the model.

The static deformation of the table is caused be gravity forces. It manifests itself as a contribution to out of flatness error. That means it can be handled on a similar way.

The largest deformations of the metrology frame are thermally induced. The main sources of the thermal disturbance are:

- heating and cooling source in the environment, like lighting, air conditioning, people around the machine etc.,
- heat generated by the machine itself,
- thermal memory: heat stored in the machine components from a previous thermal state.

The compensation of thermally induced errors is rather cumbersome, because of the complexity of the problem [13]. Based on results from the literature a linear thermal compensation model can be used.

# 3 Geometric Error Model

A coordinate measuring machine is a multiaxis machine consisting of a chain of translational and/or rotational kinematic components. The geometric deviation of a CMM is originating from the geometric deviations of its components. In order to discuss a general model the error model of the components are discussed.

The measuring loop of a coordinate measuring machine in general is given in Figure 1. Depending on the configuration of the machine all measuring motions are performed by the probe or by the table, or divided between the two.

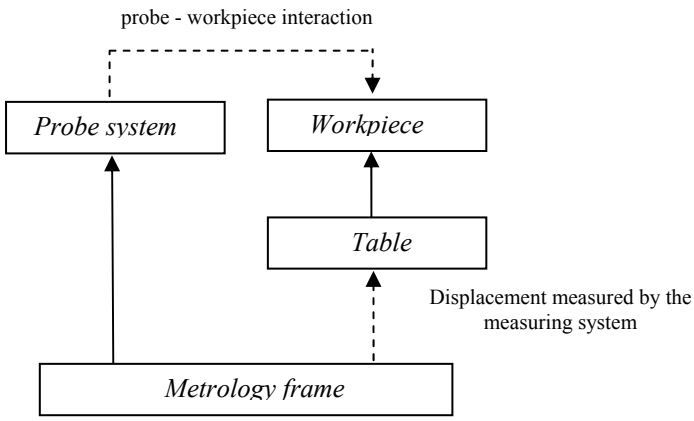



Figure 1
Schema of the metrology loop

A linear stage of precision machinery is expected to travel along a straight line and stop at a predefined position. However in the practiced the actual path deviates from the straight line due to the geometric errors of the guideways and it results also in angular errors as it is given in Fig. 2.

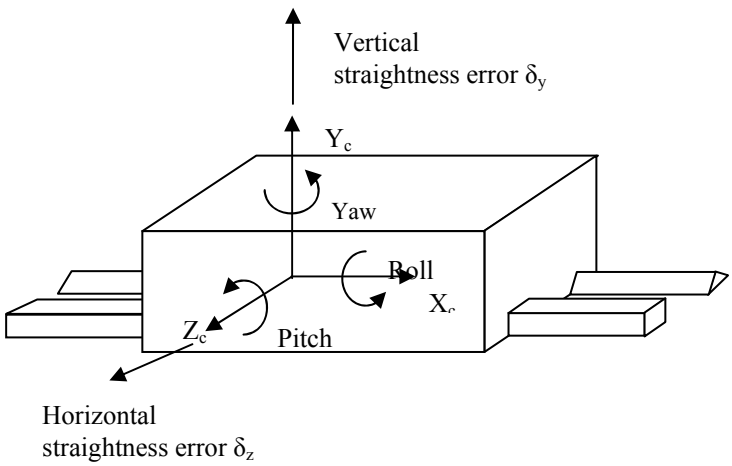



Figure 2
Representation of the six deviations of a translational kinematic component

For each axis a transformation matrix can be used to describe in homogeneous co-ordinates the deviations from the ideal motion. The general form of a transformation is given by:

$$T_{err} = \begin{bmatrix} c(\theta_y)c(\theta_z) & -c(\theta_y)s(\theta_z) & s(\theta_y) & \delta_x \\ c(\theta_x)s(\theta_z)+s(\theta_x)s(\theta_y)s(\theta_z) & c(\theta_x)c(\theta_z)-s(\theta_x)s(\theta_y)s(\theta_z) & -s(\theta_x)c(\theta_y) & \delta_y \\ s(\theta_x)s(\theta_z)-c(\theta_x)s(\theta_y)c(\theta_z) & s(\theta_x)c(\theta_z)-c(\theta_x)s(\theta_y)s(\phi_z) & c(\theta_x)c(\theta_y) & \delta_x \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where δx, δy and δz are the translational and $\theta_x$, $\theta_y$ and $\theta_z$ are the rotational components and s respectively c are short for sin and cos.

In case of the coordinate table the angular errors are very small, and all the errors are position dependent the following approximation can be made:

$$T_{err} = \begin{bmatrix} 1 & -\theta_z(x) & \theta_z(x) & \delta_x(x) \\ \theta_z(x) & 1 & -\theta_x(x) & \delta_y(x) \\ -\theta_y(x) & \theta_x(x) & 1 & \delta_z(x) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Analog results can be derived for the y and z axis. Rotational components can presented on the same way and results in rather similar matrix.

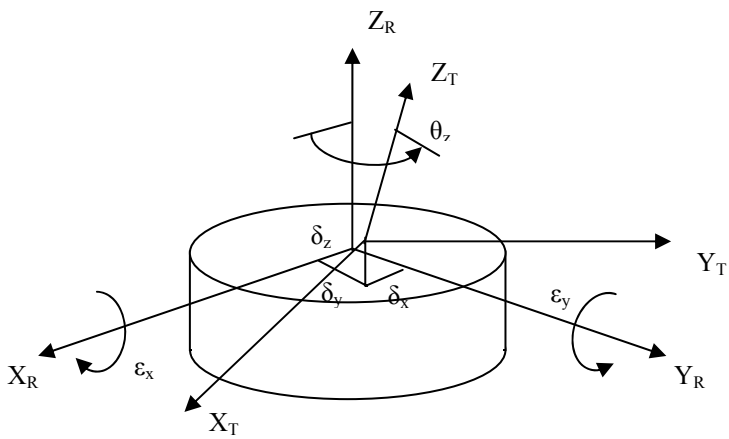



Figure 3
Representation of the six deviation of a rotational kinematic component

These components are called guided element. The guided moving elements are linked by socalled connecting elements, which can be represented by matrices with similar structure with only constant elements.

The resulting error matrix can be obtained by multiplying the individual matrices in the sequence as they follow each other in the kinematic chain.

A traditional co-ordinate measuring machine consists of three translational components x, y and z, and a probe is attached to the end of the z component. Usually the probe can be considered as a constant translational transformation.

In case of a measuring probe its error components can be handled on the similar way as it was done in case of a carriage and a rotational element.

# 4 Errors of a Coordinate Measuring Machine with three translational components

In order to illustrate the application of the technique described in the previous paragraph, let us consider a coordinate measuring machine with three translational components, given in Fig. 4.

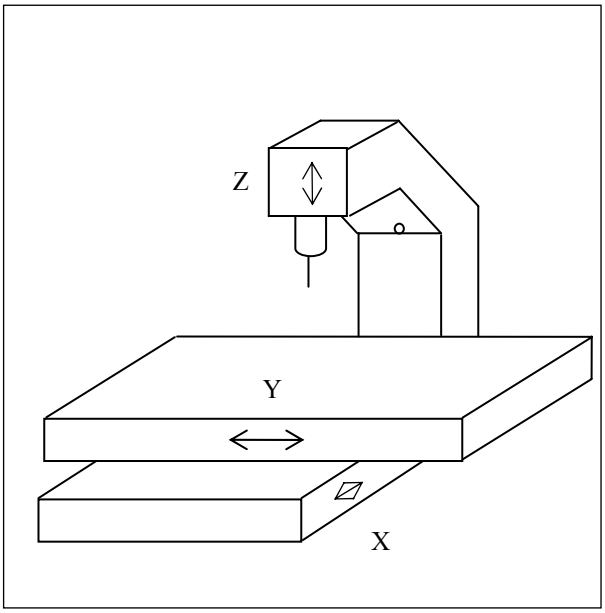



Figure 4
The investigated coordinate measuring machine

The carriage consists of two stacked tables; each of them turn rests on vacuum preloaded air bearings. The position are determined by incremental two optoelectronic measuring system, having 0,05 μm resolution. This results in a lightweight construction, which in turn ensures fast and accurate positioning.

The probe is attached to a pinole running in an air bushing. A counter weight minimizes the force needed for lifting the probe. The displacement is measured with a linear optoelectronic scale having a resolution of 0,05 μm.

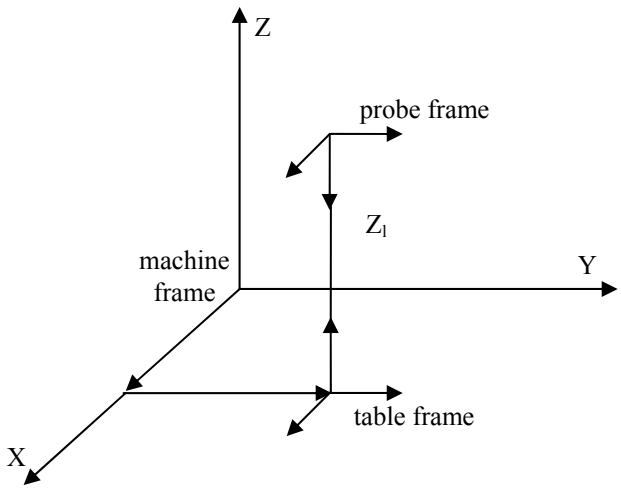



Figure 5
The frames of the CMM

The position dependent transformation matrices for the table ($T_{table}$) and the probe ($T_{probe}$) coordinate frames are given below:

$$T_{table}=\begin{bmatrix} c(\theta_y)c(\theta_z) & -c(\theta_y)s(\theta_z) & s(\theta_y) & \delta_x \\ c(\theta_x)s(\theta_z)+s(\theta_x)s(\theta_y)s(\theta_z) & c(\theta_x)c(\theta_z)-s(\theta_x)s(\theta_y)s(\theta_z) & -s(\theta_x)c(\theta_y) & \delta_y \\ s(\theta_x)s(\theta_z)-c(\theta_x)s(\theta_y)c(\theta_z) & s(\theta_x)c(\theta_z)-c(\theta_x)s(\theta_y)s(\phi_z) & c(\theta_x)c(\theta_y) & \delta_x \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & X_c \\ 0 & 1 & 0 & Y_c \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{probe}=\begin{bmatrix} c(\varphi_y)c(\varphi_z) & -c(\varphi_y)s(\varphi_z) & s(\varphi_y) & \varepsilon_x \\ c(\varphi_x)s(\varphi_z)+s(\varphi_x)s(\varphi_y)s(\varphi_z) & c(\varphi_x)c(\varphi_z)-s(\varphi_x)s(\varphi_y)s(\varphi_z) & -s(\varphi_x)c(\varphi_y) & \varepsilon_y \\ s(\varphi_x)s(\varphi_z)-c(\varphi_x)s(\varphi_y)c(\varphi_z) & s(\varphi_x)c(\varphi_z)-c(\varphi_x)s(\varphi_y)s(\varphi_z) & c(\varphi_x)c(\varphi_y) & \varepsilon_x \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & X_c \\ 0 & -1 & 0 & Y_c \\ 0 & 0 & -1 & Z_{ref}-Z_l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here again δx, δy and δz and $\varepsilon_x$, $\varepsilon_y$, $\varepsilon_z$ stand for the translational and $\theta_x$, $\theta_y$ and $\theta_z$ and $\varphi_x$, $\varphi_y$, $\varphi_z$ for the rotational errors of respectively the table and the probe. The constant values $X_c$, $Y_c$ and $Z_c$ are the offset coordinate distances between the machine coordinate system and the coordinate system attached to the workpiece to be measured. $Z_{ref}$ is the probe reference point and $Z_l$ is the probe length.

The coordinate values captured by the CMM are:

$$T_{meas} = \begin{bmatrix} x_{meas} \\ y_{meas} \\ z_{meas} \\ 1 \end{bmatrix} = T_{table} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} + T_{probe} \begin{bmatrix} x \\ y \\ z \\ y \end{bmatrix}$$

If for accuracy reason one may not substitute α for sinα and 1 for cosα than the components of each matrix describing the transformation should be captured in simultaneously. In the subsequent paragraph a suitable method is outlined.

# 5 Determination of the Geometric Errors by Measurement

For the calibration of coordinate measuring machines Zhang et al. [23] proposed to determine the angular errors by measuring the displacement errors along two parallel lines to the axis of motion but separated by a distance in the appropriate orthogonal direction.

In a more recent paper Zhang and Fu [24] describe the calibration of optical CMM-s using an uncalibrated reversible grid plate in three positions. In the initial position the plate is aligned with the machine coordinate system Next it is reversed about Y axis of the machine. In the third position the grid is rotated 90º about the Z axis. To determine the scale error one of the machine axis should be calibrated by a laser interferometer.

A simple measuring technique was invented by Fan et al. [3] to determine the motion accuracy of a linear stage. The idea is based on the fact that the position and orientation of a rigid body can be determined by appropriately selected six points. They measure the displacement of these points and calculate from them the rotational and translational error components. The displacement in the motion direction and the angular errors (pitch and yaw) perpendicular to this direction are measured by three laser interferometers. The roll and the straightness errors are captured by an optical setup containing two quadrant photo detectors. Taking again into considerations that the angular errors are small their values are replaced by their tangent. The invention initiated the development a dual and triple beam interferometers.

The above mentioned authors published a paper [5] about the measurement to determine the accuracy of a high precision wafer stage. Therefore 6-DOF errors of its positioning accuracy are significant. An improved version of the above described system was used. The moving part of it is an L-shaped mirror and on top of one leg a long right angle mirror. The stationary part consists of four laser

heads, two beam splitters and two quadrant photo detectors. The laser heads use four laser Doppler scales, three of which are parallel to each other. The upper two laser beams can be reflected by the long right-angle mirror and the lower one by the Y leg of the L shaped plane mirror. The fourth laser beam is aligned in the X-axis and reflected by the X mirror. Comparing the four linear measurements by four laser doppler scales, the X and Y positioning error of the moving table and its pitch and yaw errors can be determined. The upper two reflecting beams are split and each split beam is received by a quadrant photo detector. Comparing these signals the vertical straightness and the roll error can be derived at the same time. In order to minimize the cosine errors among the three displacement measurement and to ensure angular accuracy of the pitch and yaw measurements the parallelism of these beam should be precisely adjusted. The system takes into consideration the squareness alignments and the flatness error of the plane mirrors.

In their paper Gao et al. [7] describe the measurement straightness and rotational error motions of a commercially available linear airbearing stage actuated by a linear motor. The pitch and yaw errors were measured by an autocollimator. For the roll error measurement two capacitive displacement probes scan the flat surface in the XZ plane. The probes with their sensing axis in the Y direction were aligned with a certain spacing. The roll error is obtained by dividing the difference of the outputs of the two probes by the spacing between them. The horizontal and vertical straightness errors were measured by using the straightness kit of a laser interferometer.

The setup to detect motion errors of the linear stage uses two laser interferometers [16] and three capacitive sensors [14] is given in Fig. 4. The stationary part consists of a single and a dualbeam laser interferometer and three capacitive sensors perpendicular to each other. Both translational and rotational errors can be derived out of the displacement values captured by the transducers.

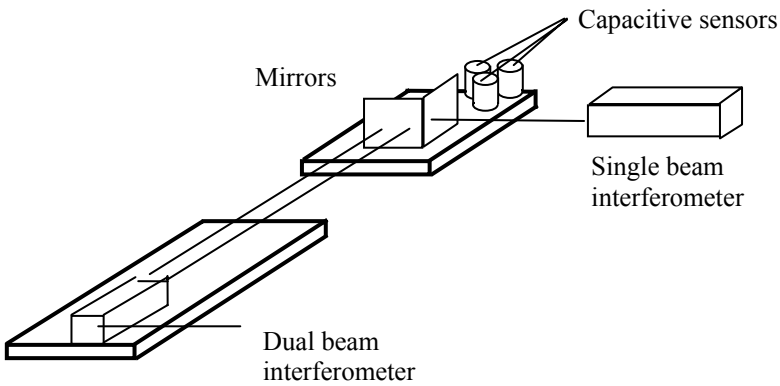



Figure 6
Set up for determining the motion error of a linear stage

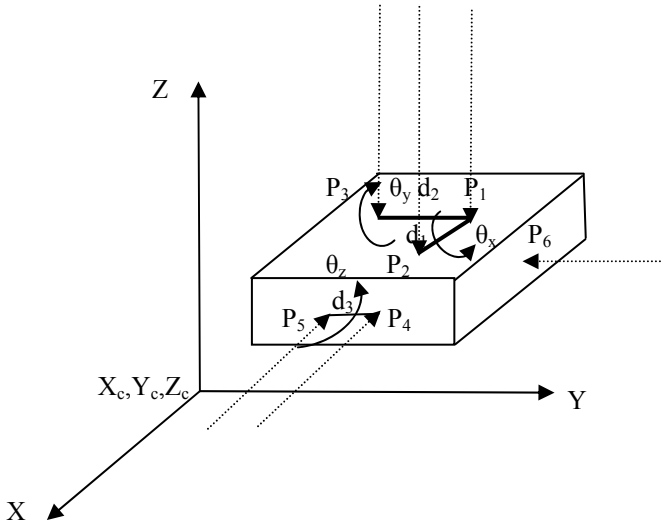



Figure 7
The measuring points on the surface of the artifact and their relations to each other

Where $d_1$, $d_2$ and $d_3$ represent the distance between the laserbeams, respectively the capacitive sensors, parallel to the coordinate axis.

By expanding the following determinants the equations of the table's boundary planes can be determined. These equations can be used to compute the origo of the new coordinate system and its principal axis. Having these values one can directly draw up the transformation matrix.

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ d_1 & 0 & d_1 tg\,\theta_y \\ 0 & d_2 & d_2 tg\,\theta_x \end{vmatrix} = 0 \qquad \begin{vmatrix} x - x_4 & y - y_4 & z - z_4 \\ d_3 tg\,\theta_z & d_3 & 0 \\ tg\,\theta_y & tg\,\theta_x & -1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - x_6 & y - y_6 & z - z_6 \\ tg\,\theta_y & tg\,\theta_x & -1 \\ -1 & tg\,\theta_z & tg\,\theta_z tg\,\theta_x - tg\,\theta_y \end{vmatrix} = 0$$

# 6 Error Compensation Scheme

The error matrices are captured at discrete points of the measuring volume. To be able to model the behavior of the machine in between the matrices should be interpolated. Componentwise interpolation leads to a non-orthonornal rotational

part which should be avoided. We may consider these matrices as a Lie group and try to do the interpolation using techniques from their theory. However this is a tedious job. Let us decompose the matrix into a rotational part and a displacement vector describing the motion of the machine components as follows:

$$T = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \delta_x \\ \delta_y \\ \delta_z \end{pmatrix}$$

Here the position dependent displacement vector points can be interpolated by a piecewise polynomial space curve while the rotational matrix remains to be interpolated. However there are simple techniques available based on the application of quaternion.

Quaternion [1] was invented by Sir William Hamilton in 1843. He realized that four numbers are needed to describe a rotation followed by a scaling. One number describes the size of scaling, one the number of degrees to be rotated and the lst two numbers give the plane in which the vector should be rotated. Quaternions consist of a scalar part $s \in \mathbb{R}$ and $\mathbf{v} = (x,y,z) \in \mathbb{R}^3$:

$$q \equiv [s,\mathbf{v}] \equiv [s,(x,y,z)] \equiv s + \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$$

where

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1,\ \mathbf{ij} = \mathbf{k} \text{ and } \mathbf{ji} = -\mathbf{k}$$

If q is a quaternion with $q = [\cos\theta, \sin\theta\mathbf{n}]$ and p is a quartenion $p = [0,\mathbf{r}]$ then $p' = qpq^{-1}$ is p rotated $2\theta$ about the axis $\mathbf{n}$.

Given a transformation matrix M the corresponding unit quaternion is can be calculated in two steps: first we must find s which is equal to:

$$s = \pm \frac{1}{2}\sqrt{M_{11} + M_{22} + M_{33} + M_{44}}$$

Now the other values follow:

$$x = \frac{M_{32} - M_{23}}{4s}$$

$$y = \frac{M_{13} - M_{31}}{4s}$$

$$z = \frac{M_{21} - M_{12}}{4s}$$

Where the M-s are the determinants of the respective submatrices.

A so-called spherical linear quaternion interpolation (Slerp) can used to compute the intermediate quaternions. The quaternions generated by Slerp are unit

quaternions, which means that they represent pure rotation matrices. The formula for Slerp is:

$$\cos(\Omega) = q_o \bullet q_1$$

$$Slerp(q_0, q_1, h) = \frac{q_0 \sin((1-h)\Omega) + q_1 \sin(h\Omega)}{\sin(\Omega)}$$

where $\bullet$ stands for the inner product defined as $q \bullet q' = ss' + xx' + yy' + zz'$

An even better (smoother) interpolation can be formulated which is the spherical cubic equivalent of a Beziér curve. This is called Squad and this defined by:

Let $q_1, ..., q_n$ point on the unit sphere. Find the cubic spline which interpolate the points in the given sequence. This can be achieved by the following formula:

**Squad** ($q_i$, $q_{i+1}$, $s_i$, $s_{i+1}$, h) = **Slerp**(**Slerp**($q_i$, $q_{i+1}$, h), **Slerp**($s_i$, $s_{i+1}$, h), 2h(1-h))

where $s_i$ are

$$s_i = q_i \cdot \exp\left(-\frac{\log \cdot (q_i^{-1} q_{i+1}) + \log \cdot (q_i^{-1} q_{i-1})}{4}\right)$$

where logq and exp q are defined as follows:

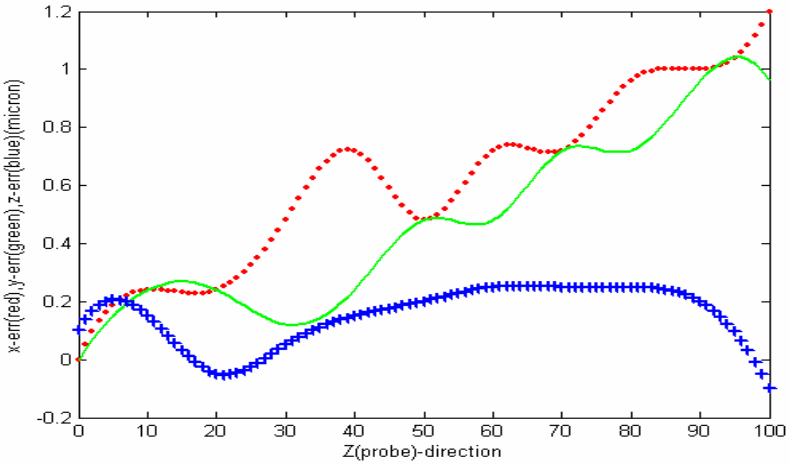
if q = [cosθ,sinθ**v**] then logq ≡ [0,θ**v**] and if q = [0,θ**v**] then expq ≡ [cosθ,sinθ**v**]

The suggested procedure to find the intermediate rotation matrices consists of the following steps:

- Convert the matrices captured by the procedure described in paragraph 4. in quaternions
- Find the **Squad** interpolation of these points
- Convert the quaternion splines back into matrix form

On this way a spherical spline representing pure rotation of the object will be obtained. The translational error can be handled by finding a spatianel interpolation spline function using for example least square fit. In the possession of these functions one can easily reconstruct the real coordinate values of the measured point.

In most of the practical applications, due to the small angular errors spherical linear interpolation is sufficient. In these cases the position originating from straightness and perpendicularity errors are dominant.




In Figure 8 the components of the error vector calculated on the above described way are given for the measuring machine presented in paragraph 4. Similar results are available for the x and the y axis.

## Conclusions

The paper presents a new approach to the compensation of geometric errors in coordinate measuring machines. It consists of a measuring procedure which captures simultaneously the six error components of moving rigid body. The transformation matrices obtained on this way are interpolated by using quaternion representation. Hereby the orthonormality of the rotation matrices are maintained. Simulation values with randomly generated error components showed that the intermediate values lead to accuracy improvement.

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