

Approximate Method for Determining the Axis of Finite Rotation of Human Knee Joint

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Abstract: The aim of this paper is to present an approximate method for determining the position and orientation of the axis of finite rotation with regard to human knee joint. The method includes data acquisition of anatomical angles and landmarks, which are considered as known inputs. Three position components of the origin of reference frame (Ot), secured to the sensor in the absolute coordinate system, and three Euler-angles between the reference frame and the absolute coordinate system are needed for the calculation. By the use of basic vector and quaternion theory, the determination of the axis of finite rotation can be carried out.

Keywords: axis of finite rotation; Euler-parameters; optical positioning; rotation

1 Introduction

Relative motion is described as a motion observed from or referred to some material system constituting a frame of reference between points or bodies. The practical use of this kind of description appears in many examples starting from a trivial, everyday event when a motorboat in a river is moving amidst the river current, up to challenging and relevant engineering questions such as the movement (and control) of non-holonomic wheeled robots [1, 2].

In biomechanics, relative motion is used for determining/measuring anatomical kinematic parameters such as rotation or ad/abduction. A widely accepted and applied contemporary method is the insertion of markers (pins, screws) into bones

[3, 4, 5], when the actual change of the motion can be revealed by observing the changes of the marker coordinates between successive locations. The determination of the axis of finite rotation can be understood as a basis for making a definite connection between the observed kinematical quantities (rotation, ad/abduction, etc) and the body segments.

Several methods are used for such determination, like the Plücker lines [6], dual vector method [7] or the instantaneous helical axis (IHA) approach [8], which is also recommended by the International Society of Biomechanics. These elaborated methods naturally have advantages and disadvantages. The Plücker lines method is sensitive to the noisy data, and encounters problem if the rotating angles are small or zero. The IHA approach is mostly used in case of neck [9], spine [10] or shoulder analysis [11], and a mention must be made that many studies, where joint kinematics is discussed, assume that the investigated motions are planar. Common disadvantage of the IHA method, likewise the Plücker lines method, that it is also quite sensitive to low angular velocities and landmark measurement errors [8, 9].

This approach employs the advantageous attributes of the Euler-parameters, and in addition, it is not restricted to planar motion, while it stays perfectly stable in case of low angular velocities. The method requires the following data: the Euler-angles and the position coordinates of the reference frame, secured to the moving part. These quantities are observed in the absolute coordinate-system.

Thus in summary, the proposed method is suitable to determine the motion of a relative coordinate system (fixed to the moving tibia) in a steady coordinate system fixed to the femur. The motion of the tibia, together with the relative coordinate system, is a combination of the local movements (rotation, ad/abduction, translation) of the knee joint. The motion, with special regard to the determination of the axis of finite rotation, is approximated by solely rotations of appointed axes, where both the position and the orientation of this moving (relative) coordinate system is to be determined.

The study first introduces how the data acquisition of the landmarks/rotation takes place with the Polaris optical tracking system, and then it is followed by the explanation of the proposed approximate method for determining the position and orientation of the finite rotation axis under human movements. In this study no numerical or experimental results are presented, it is solely restricted to the method.

2 Method

2.1 Test Equipment, Landmarks and the Demanded Quantities

The research group continuously carries out experimental tests on cadaver knees in order to create a working and acceptable prosthesis rating (qualification) method [12, 13]. Therefore, a special test equipment has been assembled, which is suited to clamp cadaver knee joints (Figure 1). This test equipment is adequate for kinematical measurements and also to provide necessary inputs (Euler-angles, position coordinates of the reference frame secured to the moving part) for the presented calculation method.

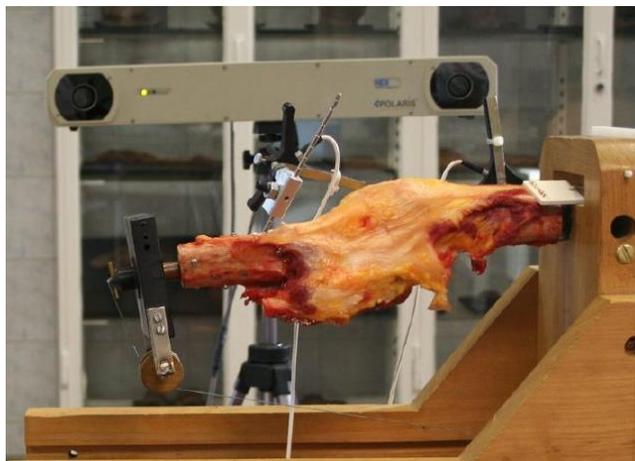


Figure 1

Experimental equipment with Polaris optical tracking system

The equipment at issue has the unique feature that the cadaver knee joint (or prosthesis) can carry out unconstrained flexion and extension without altering the non-pathological rotation or the ad/abduction of the joint. Polaris optical tracking system [14] was used for data acquisition during the flexion-extension motion. Human cadaveric knee specimens were used and two trackers were secured rigidly both to the femur and to the tibia. Via this experimental setup, the change of Euler angles (Azimuth (Ψ), Elevation (Θ) and Roll (Φ) further discussed in 2.2.3) and several anatomical landmarks can be directly recorded. Landmarks were applied according to the description of the VAKHUM project [15] as follows (Figure 2):

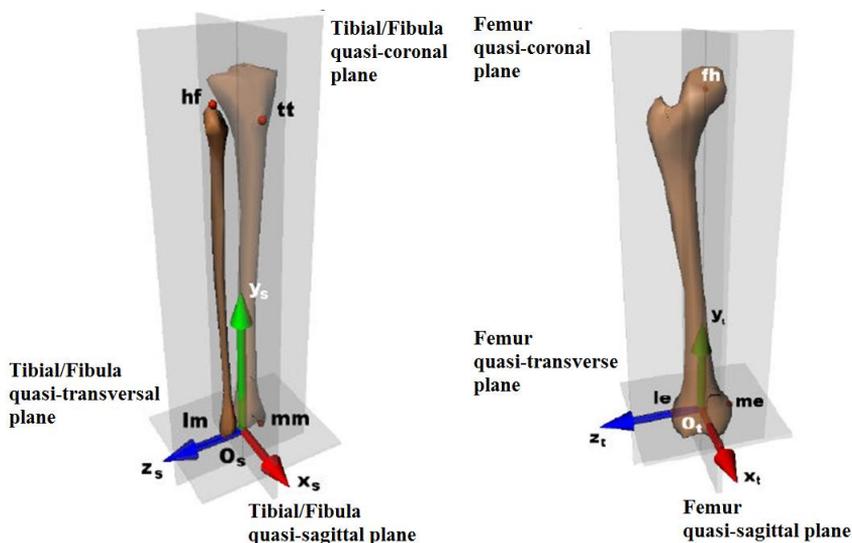


Figure 2

Anatomical landmarks defined by the VAKHUM project [15]

- Coordinates of centre of the femoral head (f_h),
- Coordinates of medial and lateral epicondyles (m_e , l_e),
- Coordinates of apex of the head of the fibula (h_f),
- Coordinates of prominence of the tibial tuberosity (t_t),
- Coordinates of distal apex of the lateral and medial malleolus (l_m , m_m),
- The origin (O_t) of the anatomical coordinate system, which is the midpoint of the junction-line between the medial (m_e) and lateral (l_e) epicondyles,
- The y_t axis of the coordinate system, which is the line between the origin and the centre of the femoral head (f_h), pointing upward with positive direction,
- The x_t axis of the coordinate system is perpendicular to the quasi-coronal plane, defined by the three anatomical points (h_f , m_e , l_e). It has positive direction to the anterior plane,
- The z_t axis of the coordinate system is mutually perpendicular to the x_t and the y_t axis with positive direction to the right.

The above mentioned parameters can be directly measured, thus they are considered as known quantities. It is worthy to note that the intra- and inter-observer variability of these landmarks is also accessible in the relevant literature [16]. Based on these quantities, more specifically the on the orientation of the relative (x_t , y_t , z_t) coordinate system and point O_t , this paper sets the accent only on determination of the axis of finite rotation under human motion.

2.2 The Approximation Method

The determination of the finite axis of rotation, as it has been discussed in the Introduction, has several different approaches. In this section, a new and simple method is introduced, where the applicable theory, and background mathematics is also discussed. The method uses formulations from different mathematical fields, while it is assembled into a series of steps that provide an approximate solution for the demanded quantities.

2.2.1 Euler Parameters – Quaternions

The application of Euler-parameters, as generalized coordinates, is not the most usual approach since it involves the concept of quaternion, however it is a widely used theory in calculations which involve three-dimensional rotation [17, 18]. A simple physical interpretation of the Euler-parameters can be seen in Figure 3.

The Euler-parameters are able to determine the orientation of the xyz reference frame secured to the moving rigid body in the steady, XYZ reference frame. If the origins of the two reference frames are coincident then the transformation is a simple rotation around the axis of revolution according to the Euler theorem [19].

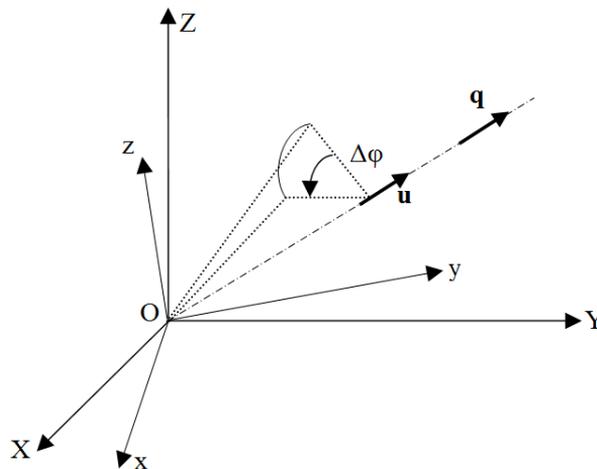


Figure 3

Transformation as a rotation around an axis

In Figure 3, the direction of the axis of revolution is denoted by \mathbf{u} unit vector, while the rotation is denoted by $\Delta\varphi$. Let us define the vector \mathbf{q} as follows:

$$\mathbf{q} = \mathbf{u} \sin \frac{\Delta\varphi}{2} \quad (1)$$

The components of vector \mathbf{q} are q_1, q_2, q_3 . By introducing

$$q_0 = \cos \frac{\Delta\varphi}{2} \quad (2)$$

quantity, the Euler-parameters, quaternion, are obtained as:

$$\mathbf{p} = [q_0, q_1, q_2, q_3]. \quad (3)$$

The quaternion includes four real elements, where the first element (q_0) is a scalar value, while the other elements (q_1, q_2, q_3) are the elements of a spatial vector. The elements of quaternion \mathbf{p} are the Euler parameters, which are equal to:

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \quad (4)$$

2.2.2 General Coordinates of a Rigid Body

Six general coordinates are required to determine the position of a rigid body in any given coordinate-system. The position of the origin of the coordinate-system is secured to the body in motion (Figure 4). The position of the origin is described by three translational coordinates.

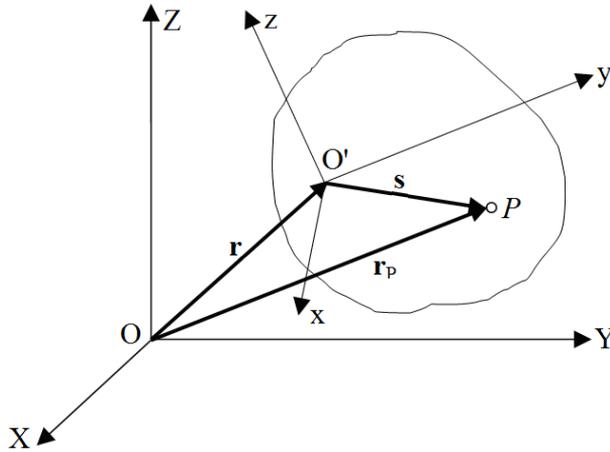


Figure 4

Determination of point P on the surface of the rigid body

The orientation of the axes of reference frame xyz to the XYZ axes is described by three additional, general rotational coordinates. If the general coordinates of the body are known during the motion, then the coordinates of point P (Figure 4) on the body can be described in the XYZ reference frame as follows:

$$\mathbf{r}_p = \mathbf{r} + \mathbf{s} = \mathbf{r} + \mathbf{A} \cdot \mathbf{s}' \quad (5)$$

Where,

$\mathbf{r}_P = [X_{0P}, Y_{0P}, Z_{0P}]^T$: Vector, pointing from the origin of the absolute system to a defined point P on the moving body. The vector is defined in the absolute system.

$\mathbf{r} = [X_{00}, Y_{00}, Z_{00}]^T$: Vector, pointing from the origin of the absolute system to the origin of the relative (moving) coordinate system. The vector is defined in the absolute system.

$\mathbf{s} = [X_{0P}, Y_{0P}, Z_{0P}]^T$: Vector, pointing from the origin of the relative (moving) system to a defined point P on the moving body. The vector is defined in the absolute system.

The rotational transformation (6) from the moving to the fixed coordinate-system can be carried out by a transformational matrix, denoted by \mathbf{A} :

$$\mathbf{s} = \mathbf{A} \cdot \mathbf{s}' \quad (6)$$

Where $\mathbf{s}' = [x_{0P}, y_{0P}, z_{0P}]^T$ is the same vector, defined in the relative (moving) coordinate-system, and \mathbf{A} is the transformational matrix expressed by the Euler-parameters [19]:

$$\mathbf{A} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_0q_3) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\ 2(q_1q_3 + q_0q_2) & 2(q_2q_3 - q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

2.2.3 Transformation of the Points by Euler Angles

Polaris optical tracking system was used for the measurements. The following kinematical variables were collected by the system: $XO, YO, ZO, \Psi, \Theta, \Phi$ (Fig. 5). The coordinates of O_t (XO, YO, ZO) are the position coordinates of the origin of reference frame, secured to the moving body, in the absolute coordinate-system.

The Euler-angles are defined as a sequence of angles (Azimuth, Elevation and Roll) that determine the orientation of the moving body with respect to the XYZ steady reference frame. Azimuth (Ψ) is a rotation of the X and Y coordinates around Z-axis. Elevation (Θ) is a rotation of the Z and the rotated X coordinates around the rotated Y-axis. Roll (Φ) is a rotation of the rotated Y and Z coordinates around the rotated X-axis (Figure 5).

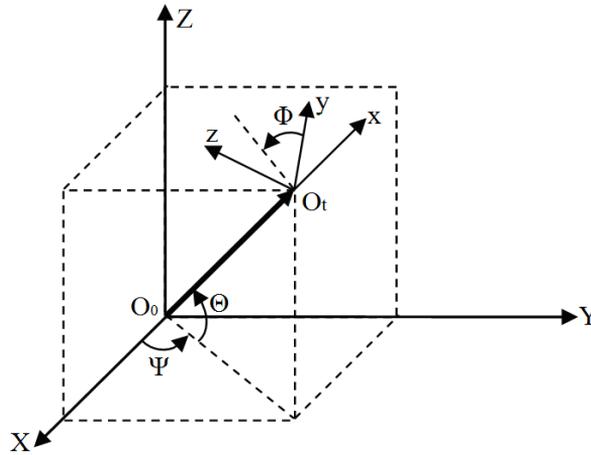


Figure 5

Definition of kinematical variables ($X, Y, Z, \Psi, \Theta, \Phi$) recorded by Polaris optical tracking system

The moving (relative) body-fixed system (xyz) can be rotated and displaced to the fixed (absolute) reference frame (XYZ) by a simple transformation (\mathbf{T}^{-1}) as follows:

$$\mathbf{x}_a = \mathbf{T}^{-1} \cdot \mathbf{x}_r \quad (7)$$

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & 0 \\ T_{21} & T_{22} & T_{23} & 0 \\ T_{31} & T_{32} & T_{33} & 0 \\ X_o & Y_o & Z_o & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (8)$$

The elements of the transformation matrix (\mathbf{T}^{-1}) can be determined with the Euler angles.

$$\mathbf{T}^{-1} = \begin{bmatrix} \cos \Theta \cdot \cos \Psi & \cos \Theta \cdot \sin \Psi & -\sin \Theta & 0 \\ -\cos \Phi \cdot \sin \Psi + \sin \Phi \cdot \sin \Theta \cdot \cos \Psi & \cos \Phi \cdot \cos \Psi + \sin \Phi \cdot \sin \Theta \cdot \sin \Psi & \sin \Phi \cdot \cos \Theta & 0 \\ \sin \Phi \cdot \sin \Psi + \cos \Phi \cdot \sin \Theta \cdot \cos \Psi & -\sin \Phi \cdot \cos \Psi + \cos \Phi \cdot \sin \Theta \cdot \sin \Psi & \cos \Phi \cdot \cos \Theta & 0 \\ X_o & Y_o & Z_o & 1 \end{bmatrix}^{-1}$$

2.2.4 Determination of Position-Orientation of Finite Rotation Axis

In this section it is shown how the transformation takes place between two, consecutive points, which is followed by the determination of the axis of finite rotation.

In Figure 6, two consecutive positions of the origin (O_i) of the anatomical coordinate system are shown. The point O_i is fixed to the tibia (moving xyz

reference frame) and it is defined in the absolute coordinate ($X_oY_oZ_o$ reference frame) system fixed to the femur. Let us denote these positions as 1^{st} and 2^{nd} positions. $\Delta\mathbf{s}_{(1,2)}$ vector describes the displacement between two consecutive points (O_{t1} and O_{t2}), and it is simple determined by the spatial Pythagoras theorem. $\mathbf{e}_{rot(1,2)}$ denotes the unit vector of the rotation axis (note that $\mathbf{e}_{rot(1,2)} = \mathbf{u}$), while $\mathbf{e}_{\Delta s(1,2)}$ describes the unit displacement vector between two consecutive points ($\mathbf{e}_{\Delta s(1,2)} = \Delta\mathbf{s}_{(1,2)} / |\Delta\mathbf{s}_{(1,2)}|$).

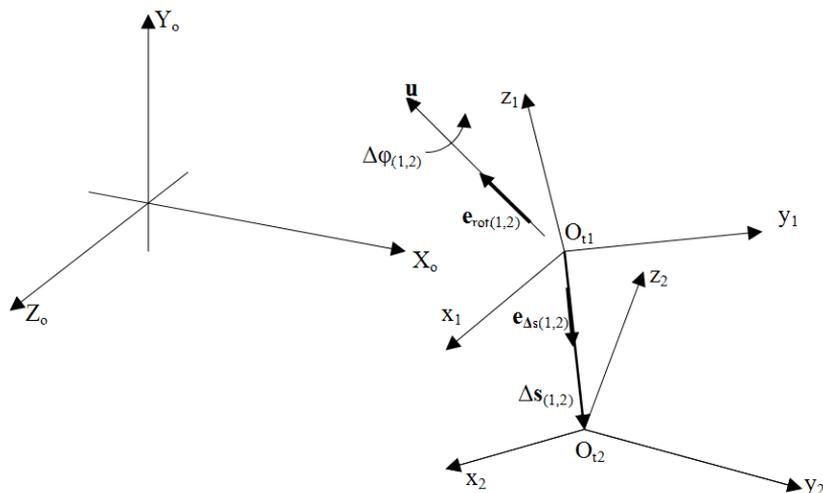


Figure 6

Consecutive positions-orientations of the moving (x,y,z) reference frame in the absolute ($X_oY_oZ_o$) frame

The reference frame of the sensor is secured to the moving tibia. It must be noted that the demonstrated positions, in reality, are located relatively close to each other, therefore the distance between the origins is not more than a few mm.

The relation between the first and second moving positions (\mathbf{T}_{1-2}) of the tibia can be derived by the following matrix-equation:

$$\mathbf{T}_{1-2} = \mathbf{T}_2 \cdot \mathbf{T}_1^{-1}, \quad (9)$$

where \mathbf{T}_2 is the transformational matrix of the second position (identified in the Polaris space), while the \mathbf{T}_1^{-1} is the inverse matrix of the first position.

The third order sub-matrix of the \mathbf{T}_{1-2} and the rotational matrix of \mathbf{A} are kinematically equivalent in the aspect of the two positions of the tibia. Due to this coequality, the Euler-parameters can be calculated. The elements of matrix \mathbf{A} are the Euler-parameters that determine the revolution. In details: they determine the unit vector of the axis of revolution and the angle of rotation, with correct algebraic sign, of the rigid body. In our current case, this is the revolution of the tibia around a fixed axis.

The requested angle of rotation with regard to the Euler-parameters:

$$\Delta\varphi = 2 \cos^{-1} q_o, \quad (10)$$

While the components of the unit vector, parallel to the unit vector of the axis of revolution:

$$\mathbf{u} = \left[\begin{array}{c} q_i \\ \sin \frac{\Delta\varphi}{2} \end{array} \right] \quad i = 1,2,3 \quad (11)$$

The following question has to be answered: what is the position and orientation of the finite rotation axis between two positions in the moving reference frame?

The calculation of the position and orientation of the finite axis of rotation can be accomplished in two steps (Figure 6):

- If the origins of the moving reference frame (O_{i1} and O_{i2}) are coincident, then the rotation can be determined by the Euler-parameters, more specifically, by the angular displacement $\Delta\varphi$ and the unit vector $\mathbf{e}_{\text{rot}(1,2)}$ of the rotation axis.
- The position of the finite axis of rotation is described by the displacement vector $\Delta\mathbf{s}_{(1,2)}$ between the two consecutive origins (O_{i1} and O_{i2}) and the angular displacement $\Delta\varphi$.

With regard to the modeling questions, it must be noted that several studies showed that the tibia, with respect to the femur, follows a complex one degree of freedom spatial path during passive flexion [20, 21, 22]. Another study from Sancisi et al. [23] presented that the calculation of anatomical angles (rotation, ad/abduction, etc) can be carried out, with reasonable accuracy, if simple spherical constraints are applied. Nevertheless, these simplifications must be carefully applied in the modeling, since the so-called roll-back [24] motion has also great impact on the movement and cannot be completely disregarded.

Based on the above mentioned studies, if the knee joint is considered as a spherical joint (rotation without any translation), then $\mathbf{e}_{\text{rot}(1,2)}$ and $\Delta\mathbf{s}_{(1,2)}$ vectors will be perpendicular to each other.

Based on these vectors and the proposed approximation, the position and orientation of the finite rotation axis can be determined (Figure 6 and Figure 7).

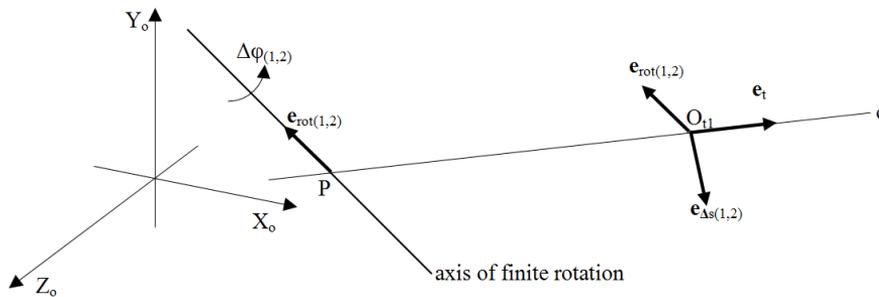


Figure 7

Determination of axis of finite rotation

The axis of finite rotation is parallel to unit vector $\mathbf{e}_{\text{rot}(1,2)}$ and located on point P (Figure 7). Point P is also located on the line c , perpendicular to a plane formed by vectors $\mathbf{e}_{\text{rot}(1,2)}$ and $\Delta \mathbf{s}_{(1,2)}$. The direction of the normal vector of the plane can be determined by vector product:

$$\mathbf{e}_t = \mathbf{e}_{\Delta s(1,2)} \times \mathbf{e}_{\text{rot}(1,2)}. \quad (12)$$

To determine the position of the finite axis of rotation, let us denote $\overline{PO_{t1}}$, the absolute distance between the axis of finite rotation and a certain point P , as t :

$$\overline{PO_{t1}} = t, \quad (13)$$

where t can be calculated as:

$$t = \frac{|\Delta \mathbf{s}_{(1,2)}|}{|\Delta \varphi|}. \quad (14)$$

Finally the coordinates of point $P (P_x, P_y, P_z)$ on the line c can be obtained from the following equations:

$$(P_x - O_{t1x})^2 + (P_y - O_{t1y})^2 + (P_z - O_{t1z})^2 = t^2, \quad (15)$$

$$\frac{P_x - O_{t1x}}{e_{tx}} = \frac{P_y - O_{t1y}}{e_{ty}} = \frac{P_z - O_{t1z}}{e_{tz}}. \quad (16)$$

Conclusions

In this paper an approximate method is presented for the determination of the position and orientation of finite rotation axis during human movements. The proposed method is not restricted to planar movement and in addition, it is independent of the angular velocity of the examined motion. The calculation can be carried out with the help of simple vector algebraic tools and with basics knowledge about quaternions.

Determination of the position and orientation of finite rotation axis can serve as a tool for several applications such as the calculation of instantaneous center of rotation regarding knee joint, functional spinal unit [25] or an alternative way to investigate the sliding-rolling phenomenon between connecting surfaces of femur and tibia [26]. This method can be applied in any kinematical investigations of human joints, regardless of the position and orientation of the coordinate-systems secured to the body segments. As for further aims, the method will be applied on the data which has been obtained from the cadaver knees, and the actual numerical results will be compared to other authors' results in order to show the accuracy and simplicity of the proposed method.

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